Applied Mathematical Modelling 89 (2021) 1433–1458

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Multi-period dynamic distributionally robust pre-positioning of emergency supplies under demand uncertainty

Ming Yang[a], Yankui Liu[b,1], Guoqing Yang[b,*]

[a] Risk Management & Financial Engineering Laboratory, College of Mathematics & Information Science, Hebei University, Baoding 071002, Hebei, China
[b] School of Management, Hebei University, Baoding 071002, Hebei, China

A R T I C L E   I N F O

Article history:
Received 20 April 2018
Revised 17 July 2020
Accepted 6 August 2020
Available online 25 August 2020

Keywords:
Disaster relief
Multi-period dynamic pre-positioning
Distributionally robust optimization
Chance constraint
Demand uncertainty
Safe approximation

A B S T R A C T

The pre-positioning problem is an important part of emergency supply. Pre-positioning decisions must be made before disasters occur under high uncertainty and only limited distribution information. This study proposes a distributionally robust optimization model for the multi-period dynamic pre-positioning of emergency supplies with a static pre-disaster phase and a dynamic post-disaster phase. In the post-disaster phase, the uncertain demands are time varying and have partial distribution information that belongs to a given family of distributions. The family of distributions is described by a given perturbation set based on historical information. Therefore, the proposed model forms a semi-infinite programming problem with ambiguous chance constraints, which typically would be computationally intractable. We refine the bounded perturbation sets (box, box-ball and box-polyhedral) and develop computationally tractable safe approximations of the chance constraints. Finally, a realistic application to the Circum-Bohai Sea Region of China is presented to illustrate the effectiveness of the robust optimization model.

1. Introduction

Currently, natural disasters (earthquakes, floods, hurricanes, tornadoes and wildfires) are increasing and affecting the safety of people's lives and property. Due to the enormous loss caused by these disasters, growing attention is being given to the pre-positioning of emergency supply problem, which belongs to the preparedness and response stages in disaster management [1,2]. The pre-positioning of emergency supply problem ensures that emergency supplies can be delivered to the affected area in a timely and effective manner to maximize the role of emergency supplies. The pre-positioning process can be divided into two states: the natural state (pre-disaster) and the emergency state (post-disaster). The key decisions for the natural state are the number and locations of facilities, as well as the allocation of the multiple emergency supplies. In the emergency state, the pre-positioned emergency supplies should be sufficient to meet all demands in the affected areas at the fastest speed and lowest cost. Since emergency supplies and budgets are limited, it is challenging to find a good trade-off between maximizing demand coverage and minimizing the total costs in the preparedness stage.

Supply delay is a major obstacle to the distribution of commodities. Studies of static emergency supply pre-positioning problems often address the commodity co-ordination of supply and demand, disregarding the time and manner of

* Corresponding author.
E-mail addresses: yangmingmath@163.com (M. Yang), yliu@hbu.edu.cn (Y. Liu), ygq@tju.edu.cn (G. Yang).
[1] Tel & Fax: +86 0312 5066629.

https://doi.org/10.1016/j.apm.2020.08.035
0307-904X/

commodity distribution. In general, the number of affected people arriving at a given safe area is unknown before the disaster and information updates happen over time during an event [3]. Thus, few studies have explored the multi-period dynamic decision problem, which determines the most beneficial manner for distributing commodities in each period based on time-variant demand. This paper develops a dynamic emergency supplier pre-positioning model that can effectively reduce the delay problem in commodities distribution.

The “multi-period dynamics” in our study include two aspects of the emergency states. First, the transportation distribution process of emergency commodities in an emergency state is designed over a planning horizon consisting of several days. During the planning horizon, the proposed model is updated each day by incorporating new information on the demand quantity and the transportation quantity. Second, because it is a long-term emergency response operation, we assign a time-variant penalty that minimizes the total waiting time. The decision maker must plan pre-disaster decisions on facilities and emergency commodities by incorporating dynamic distribution decisions. The dynamic features of emergency supplies also give feedback on the decision making in the static phase. Thus, we form a model in which emergency supplies decisions are influenced through the dynamic pre-allocation decision in the post-disaster phase.

Uncertainty in demand can fluctuate unexpectedly due to many sources. These sources include the number of casualties, damage to buildings, weather conditions and so on. Accurate demand assessment, which is related to the number of affected people, is essential for achieving accurate models and maximizing the benefits of commodities utilization. To measure demand for emergency supplies, we use a probability prediction method based on the historical data of disasters. Considering the low probability of disaster occurrence and the long-term strategy of establishing an emergency supplies warehouse, to reduce the impact of a disaster and ensure a certain economy, decisions regarding emergency supplies pre-positioning should be robust. Such robustness is crucial for long-term decisions because the decisions made at the pre-disaster phase are of a strategic type and cannot be changed easily. To address these issues, this paper studies the pre-positioning of emergency supplies problem using a distributionally robust optimization approach. The approach focuses on managing the random variables with only partial distribution information. In addition, similar to stochastic programming, the decision may not meet the constraints in a highly uncertain environment, i.e., the approach must allow for a certain degree of constraint violation. Thus, we develop an ambiguous chance constraint programming model and build a safe approximation of the chance constraint to overcome the intractable formulation.

In general, government organizations often perform all of the operations over the disaster life cycle. When a disaster occurs, there is considerable pressure to deliver emergency supplies to the affected areas immediately. Time is a scarce resource used to measure the performance of a humanitarian response to a disaster. Thus, shortages and the incorrect delivery of commodities have a crucial effect on the rescue. The dynamic emergency supply pre-positioning model can enhance emergency response capacity and preparedness for disasters. In addition, the distributionally robust optimization approach can achieve a higher degree of demand satisfaction at a lower cost under uncertain environments.

The main contributions of this paper are summarized as follows:

- This paper develops a multi-period dynamic emergency supplies problem with a static pre-disaster phase and a dynamic post-disaster phase. The post-disaster phase includes a number of periods and time-variant uncertain demand. In addition, a time-variant penalty function is introduced to describe the loss resulting from time delays for dynamic demand.
- This paper establishes a distributionally robust optimization model with ambiguous chance constraints for the pre-positioning of emergency supply problems. The proposed model involved semi-infinite programming and is typically computationally intractable. We refine the bounded perturbation sets (box, box-ball and box-polyhedral) and develop a computationally tractable safe approximation of the chance constraints.
- This paper studies a realistic world case (the Circum-Bohai Sea Region). Numerical experiments are provided to illustrate the value of distributionally robust optimization in the emergency supply problem and demonstrate the performance of the dynamic model compared with the static model.

The rest of this paper is organized as follows. In Section 2, the relevant literature is reviewed. Section 3 develops a novel dynamic pre-positioning of emergency supplies model. In Section 4, robust optimization is applied in the model under demand uncertainty. Section 5 presents numerical experiments to illustrate the robustness and effectiveness of the proposed robust optimization method. Some managerial insights are summarized in Section 6. The conclusions and future research are presented in the final section.

2. Literature review

This section presents a review of the emergency supplies pre-positioning literature to establish a framework for this research. The literature related to our problem can be categorized into three streams: the pre-positioning of emergency supplies problem, the dynamic emergency supplies problem, and uncertainty modeling in the pre-positioning of emergency supplies.

The pre-positioning of emergency supplies problem has focused mainly on the process of location with stock pre-positioning and supplies pre-allocation. The decisions of stock pre-positioning take place in the preparedness phase [4–7], while the decisions of supplies pre-allocation are focusing on the response phase [8–11]. In different application backgrounds, the decision makers often utilize various transportation modes to carry the commodities, medical staffs, and relief workers to the affected areas. Ahmadi et al. [12] formulated a mixed integer linear programming to determine the locations of local depots and vehicle routings in San Francisco city. Barbarosoglu et al. [13] proposed a hierarchical multi-criteria methodology for helicopter logistics planning to transport wounded people. Due to the damage of roads after a disaster, [14] also used helicopters to transport relief personnel and medicine at first 72 h. Most of these papers dedicated to a city or a small region. The emergency supplies pre-positioning problem is a long-term preparedness strategy to prevent the occurrence of disasters. For some countries with large populations and territories, it is impossible to pre-positioning a large facility in a small area. In our study, we try to develop a model to support decision-making for a large region. It is of practical significance to plan different size facilities in large regions or multiple cities. In addition, due to the large scale areas, railway mode is suitable for long-distance transportation.

Multi-period dynamic emergency supply problem adequately represents that the traffic network information can change over time based on different conditions. When disaster strikes, the relevant data may change unexpectedly. These changes will have an impact on the emergency plans, and it is necessary to adjust the plans incorporating new information. Thus, this problem can be modeled statically in pre-disaster phase and dynamically in post-disaster phase [15], which has been studied in several literature by optimizing different objectives, such as the expected time-to-respond [16], the unsatisfied demand and the risk [17], the total weighted response time [18] and the performance of different preparedness scenarios [19]. Rawls and Turnquist [3] studied a dynamic allocation model that optimized deliver planning for satisfying short-term demands for emergency supplies. The aim of the model was meeting the demand from distant suppliers in the first 72 h. Yi and Özdamar [20] proposed an integrated location-routing model for coordinating logistics support and evacuation operations in disaster response activities. Bozorgi-Amiri and Khorsi [21] presented a multi-objective dynamic stochastic programming model by integrating pre-disaster plans and post-disaster decisions. All these papers dedicated to short-term emergency supply, only a very small proportion took a point of view on the long-term preparedness of emergency supply. This paper aims to address medium-and-large-scale disasters that needs a long time rescue.

In emergency supply operations, the exact information on the distribution of uncertainties is not usually known in advance. Due to the unpredictability of time, place, population density and magnitude of the disaster, there exists some uncertainties, such as demand uncertainty [22,23], travel times uncertainty [24], discount factor uncertainty [25] and cost uncertainty [26], that may cause the loss of life and property and can be seen as significant penalty costs. To eliminate the risk of uncertainty, various optimization methods have been documented in the literature. Considering the uncertainty of the events location and severity, [27] presented a stochastic optimization model to plan the strategic arrangement of budget-limited supplies and assets in advance of major disasters. Mohamadi and Yaghoubi [28] proposed a bi-objective stochastic model for emergency network design with backup services in the event of a disaster interruption. Glock et al. [29] developed an economic order quantity model with fuzzy demand and were first to incorporate learning in fuzziness in an inventory model. Bai et al. [30] built a fuzzy value-at-risk model for the pre-positioning emergency supplies problem with multi-affected areas and multi-relief. The transportation cost, supply, demand and capacity were represented as fuzzy parameters with variable possibility distributions. The above stochastic and fuzzy methods are proposed to handle uncertainty under partial distribution information.

When the distribution information of uncertainty is partially known, robust optimization would be an appropriate research tool [31,32]. In recent years, robust optimization methods in the emergency supplies pre-positioning problem is a hot research topic. By considering expected uncovered demand as an objective function, [33] proposed a robust bi-level optimization model for the relief distribution network. Zokaee et al. [34] proposed a three-level robust relief chain mode in which the travel time parameters, demand, supply, and cost were subject to uncertainty. Hu et al. [35] addressed a bi-objective robust emergency resource allocation problem to maximize efficiency and fairness under different sources of uncertainty. Haghi et al. [36] developed a multi-objective robust programming model for goods and casualty logistics under demand and resource uncertainties. They attempted to maximize the response level to the medical needs of the casualties and minimize the total costs of the preparedness and response phases. Najafi et al. [37] applied robust optimization to manage the logistics of relief commodities and injured people in the post-disaster phase. Ben-Tal et al. [38] built an affinely adjustable robust counterpart approach to dynamically assigning emergency response and evacuation traffic flow planning with time-dependent demand uncertainty. In [39], a min-max robust model is proposed for the location and emergency inventory pre-positioning problem. They captured the uncertainties in both the left- and right-hand-side parameters.

There is still a gap between the conservatism of robust optimization and the specificity of stochastic programming, and solutions to uncertain optimization problems are sought for partially characterized uncertainty distributions [40]. Distributionally robust optimization [40–42,45] is suitable for this case and has been applied to many topics, such as the newsvendor problem [43], power flow [44], the lot-sizing problem [46], and the medical appointment scheduling problem [47]. Our proposed model is the first to apply distributionally robust optimization into the pre-positioning of emergency supplies problem with dynamic post-disaster decisions. The advantage of the distributionally robust model is demonstrated through a comparison with the deterministic model and the robust model using the same case study.

3. A new robust emergency supplies pre-positioning model

Pre-positioning emergency supplies is a means for increasing preparedness for natural disasters. The main goal of emergency supplies is to minimize the average response time of the pre-positioning network after a disaster and ultimately reduce the loss of life and property. Sufficient commodity reserves are necessary for timely and effective responses to sudden disasters. However, with the uncertainty in the occurrence time of disasters, the commodities may not be used for one or even several years. Thus, long-term storage may expensive in terms of routine management and wastage. Due to the limited reserve funds, many studies on emergency rescue focus on addressing small-scale emergencies, and few studies address large-scale, wide-impact, long-duration emergencies. Large-scale emergency commodity reserves require a more reasonable strategy. In this section, we develop a multi-period dynamic emergency supply pre-positioning model with a high coverage area. The model has a higher rescue capability with a certain cost.

In the pre-positioning problem, there are two phases: the pre-disaster phase and the post-disaster phase. In the pre-disaster phase, the inventory size, the location of facilities and the quantities of emergency commodities to purchase are the main decisions. Because it reflects long-term security planning, the model will minimize cost by determining the above parameters before a disaster. In the post-disaster phase, we study a dynamic emergency commodity allocation model. The objective of this phase is to minimize the transshipment, holding and unmet demand penalty cost based on the pre-positioning network within a given horizon.

To build the pre-positioning problem, the required notations are as follows:

Sets:

T: set of planning horizons;
N: set of nodes;
A: set of directed arcs;
K: set of resources of type;
L: set of different facility sizes.

Parameters:

F_{il}: the fixed cost of opening a facility of size category l at location i;
a_k: unit acquisition cost of commodity k;
c_k: unit distance cost of transporting commodity k;
d_{ik}^t: the forecast demand for commodity k at location i on day t;
h_k: unit holding cost for commodity k;
m_k: unit unmet demand penalty cost of commodity k;
ν_k: unit space requirement for commodity k;
γ_l: the overall capacity of a facility in category l;
b_k: unit weight of commodity k;
B_{ij}: the overall bearing capacity of link (i, j);
r_{ij}: distance across link (i, j).

Decision Variables:

y_{il}: a binary decision variable indicating whether a facility with size l is located at node i or not;
q_{ik}^t: amount of commodity k pre-positioned at location i on day t;
Q_{ik}: amount of commodity k pre-positioned at location i;
x_{ijk}^t: amount of commodity k shipped across the link (i, j) on day t;
w_{ik}^t: shortage of commodity k at location i on day t;
z_{ik}^t: amount of unused commodity k at location i on day t.

We consider a set of essential commodities that may be pre-positioned in storage facilities and for which there is likely to be demand after an event such as an earthquake. The commodities can be pre-positioned at location i if a storage facility of a specific size l is made available. The storage facility location and commodities stocking decisions should be made before a disaster. When a disaster occurs, the various commodities are distributed from the storage facilities to the affected areas through a transportation network to meet demand. From the above discussion, we can formulate an optimization problem as follows.

For costing purposes, the total cost of the problem includes the fixed cost, acquisition cost and transshipment cost, as well as holding cost and unmet demand penalty cost. These costs reside in two phases: the pre-disaster phase and the post-disaster phase. In the pre-disaster phase, we denote $\Sigma_{i \in N}\Sigma_{l \in L}F_{il}y_{il}$ as the fixed cost for opening a facility. In an emergency supplies pre-positioning problem, the transportation of a commodity is a pre-allocation activity. The notation q_{ik}^t represents the amount of commodity k pre-positioned at location i on day t. Then, various commodities Q_{ik} can be stocked at the facilities, where $Q_{ik} = \sum_{t\in T} q_{ik}^t$. The acquisition cost is $\Sigma_{i \in N}\Sigma_{k \in K}a_k Q_{ik}$. In the response phase, the various commodities q_{ik}^t are shipped across a transportation network to meet the demand in a given planning horizon. Let $\sum_{t\in T}\sum_{(i,j)\in A}\sum_{k\in K} c_k r_{ij} x_{ijk}^t$ be the cost of transporting commodities across link (i, j) in the transportation network. The cost of holding the commodities that are not used at locations is denoted $\sum_{t\in T}\sum_{i\in N}\sum_{k\in K} h_k z_{ik}^t$.

In one time period, it is almost unavoidable for some affected people to not receive a delivery; then these affected people will have to wait for the next delivery period to receive aid. A penalty function is induced into the model to reduce the more severe loss of life and property after a disaster. Wen et al. [48] utilized a penalty function that increases quadratically with customer waiting time. In the event of an emergency, the affected people will suffer still greater losses with longer waiting times. Thus, to minimize the total number of affected people waiting, we assign a penalty function that increases cubically with the delay time, i.e., $M_k^t = m_k(\frac{t}{T})^3$, where m_k is the unit penalty cost, and t represents the day when commodities are delivered. The penalty function tends to provide shorter waiting times for several affected people rather than longer waiting times for a few [48]. Then, the penalty cost is defined as $M_k^t w_{ik}^t$.

Based on the above description, we obtain the following objective function, which includes the cost of the pre-positioning location of facilities, the acquisition of commodities, the transportation of commodities, the holding of commodities and the unmet demand penalty of commodities.

$$\sum_{i\in N}\sum_{l\in L} F_{il}y_{il} + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K} a_k q_{ik}^t + \sum_{t\in T}\sum_{(i,j)\in A}\sum_{k\in K} c_k r_{ij} x_{ijk}^t + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K}(h_k z_{ik}^t + M_k^t w_{ik}^t). \tag{1}$$

We assume that the commodities require one day to be transported to a location. Based on this, we present the flow conservation constraint for commodity k at location i on day t.

$$\sum_{(j,i)\in A} x_{jik}^{t-1} + q_{ik}^{t-1} + z_{ik}^{t-1} - z_{ik}^t = \sum_{(i,j)\in A} x_{ijk}^{t-1} + d_{ik}^{t-1} + w_{ik}^{t-1} - w_{ik}^t, \quad \forall t\in T,\ i\in N,\ k\in K, \tag{2}$$

where the left hand side of the equation represents the total amount of commodity k shipped into location i, pre-positioned at location i and unused at location i on day $t-1$ minus the amount of commodity k unused at location i on day t, and the right hand side of the equation represents the total amount of commodity k shipped out of location i, demand at location i and shortage at location i on day $t-1$ minus the amount of the shortage in commodity k at location i on day t. The dynamic of flow conservation is that the balance of commodities is determined by their inflow and outflow and their changes at each node and on each day. Note that the shortage of commodities is zero at the initial moment (pre-disaster), i.e., $w_{ik}^0 = 0$. The unused and unsatisfied demands are important concerns of the emergency preset problem. In the actual rescue process, the amount of unused and unsatisfied commodities z_{ik}^{t-1} and w_{ik}^{t-1} still need to be overlying in the next time period. Therefore, instead of an equilibrium equation for each period of time, the decision variables of unused and unsatisfied demands have an iterative relation. According to the recursive relation, the flow conservation constraint (2) can be reformulated as

$$w_{ik}^t - z_{ik}^t = \sum_{t'=0}^{t-1}\left(\sum_{(i,j)\in A} x_{ijk}^{t'} - \sum_{(j,i)\in A} x_{jik}^{t'} + d_{ik}^{t'} - q_{ik}^{t'}\right), \quad \forall t\in T,\ i\in N,\ k\in K. \tag{3}$$

The pre-positioned commodities in facilities should not exceed the facility capacity, and the allocated commodities in the link should not exceed the arc bearing capacity. According to this situation, Constraint (4) ensures that the pre-positioned commodities stocked in facility i are limited by its capacity, which is the volume of purchased goods at any given open facility. Constraint (5) limits the amount of relief commodities in link (i, j).

$$\sum_{t\in T}\sum_{k\in K} \upsilon_k q_{ik}^t \le \sum_{l\in L}\gamma_l y_{il}, \quad \forall i\in N, \tag{4}$$

$$\sum_{k\in K} b^k x_{ijk}^t \le B_{ij}, \quad \forall t\in T,\ (i,j)\in A. \tag{5}$$

In general, only one facility is available at a candidate node, and Constraint (6) ensures that the number of open facilities at node i is one.

$$\sum_{l\in L} y_{il} \le 1, \quad \forall i\in N. \tag{6}$$

To optimize the pre-positioned network, the model may assign some commodities from one zero-commodities node to another in advance, which leads to a greater shortage at this mode. However, this process is impossible in real life because the outflow of emergency commodities at a node on day t must be determined by the pre-positioned or inflow prior to that day. To model this situation, Constraint (7) refers to the commodities flow restriction constraint, which states that it is only possible to ship a commodity from node i in period t when there is an unused quantity of commodity z_{ik}^{t-1} at location i on day $t-1$. The outflow of node i should be restricted by the difference between the unused and the unmet quantities of a commodity on the day t.

$$\sum_{(i,j)\in A} x_{ijk}^t \le z_{ik}^t - w_{ik}^t, \quad \forall t\in T,\ i\in N,\ k\in K. \tag{7}$$

Considering the realistic nature of the model, feasible regions for decision variables are enforced by the following constraints:

$$y_{il} \in (0,1), \quad \forall i\in N,\ l\in L, \tag{8}$$

$$q_{ik}^t, w_{ik}^t \ge 0, \quad \forall t\in T,\ i\in N,\ k\in K, \tag{9}$$

$$x_{ijk}^t \ge 0, \quad \forall t\in T,\ (i,j)\in A,\ k\in K. \tag{10}$$

Constraint (8) states that y_{il} equals 1 if there is a supply facility of capacity category l located at node i and 0 otherwise. The remaining constraints (9) and (10) are non-negativity conditions.

It is clear that usually when disasters occur, the exact demand information is not known in advance. In this study, we develop the uncertain formulation of the pre-positioning model with demand uncertainty. It is assumed that the uncertainty of the demand $\tilde{d}^t_{ik}$ is affinely dependent on a set of independent random variables ζ^t_{ik} taking values in given finite segments [-1,1] and following a distribution symmetric with respect to zero, i.e.,

$$\tilde{d}^{t'}_{ik} = \bar{d}^{t'}_{ik} + \hat{d}^{t'}_{ik}\zeta^{t'}_{ik}, \quad \forall t \in T,\ t' \in \{0, ..., t-1\},\ i \in N,\ k \in K,$$

where $\bar{d}^t_{ik}$ is the nominal value of demand, and $\hat{d}^t_{ik}$ represents positive constant perturbation.

Ideally, the decision maker may prefer a set of solutions that make the constraint feasible with probability 1. However, in reality, the solutions do not need to be overly conservative and can allow for a certain degree of constraint violation. This approach provides a probabilistic guarantee of robust solution feasibility when uncertain parameters are perturbed. Altering Eq. (3), we can build the following chance constraint with at least probability $1-\epsilon^t_{ik}$:

$$\Pr_{\tilde{d}^{t'}_{ik}\sim P}\left\{\sum_{t'=0}^{t-1}\left(\sum_{(i,j)\in A} x^{t'}_{ijk} - \sum_{(j,i)\in A} x^{t'}_{jik} + \tilde{d}^{t'}_{ik} - q^{t'}_{ik}\right) \le w^t_{ik} - z^t_{ik}\right\} \ge 1-\epsilon^t_{ik}, \forall t \in T,\ i \in N,\ k \in K, \tag{11}$$

where $\epsilon^t_{ik} \in (0,1)$, and P is the known probability distribution of random variables $\tilde{d}^{t'}_{ik}$. Note that, to compute (11), the random variables are usually distributed according to an exact distribution. However, in practice, it is difficult to obtain the distribution information of uncertain parameters; these usually have only partial or no information on probability distribution P, which belongs to a given family $\mathcal{P}$. For all probability distributions in $\mathcal{P}$, we can obtain the following model with the *ambiguous* chance constraint:

$$\begin{aligned}
\min_{y,q,x,z,w}\quad & \sum_{i\in N}\sum_{l\in L} F_{il}y_{il} + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K} a_k q^t_{ik} + \sum_{t\in T}\sum_{(i,j)\in A}\sum_{k\in K} c_k r_{ij} x^t_{ijk} + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K}(h_k z^t_{ik} + M^t_k w^t_{ik})\\
\text{s.t.}\quad & \Pr_{\zeta^{t'}_{ik}\sim P}\left\{\sum_{t'=0}^{t-1}\left(\sum_{(i,j)\in A} x^{t'}_{ijk} - \sum_{(j,i)\in A} x^{t'}_{jik} + \bar{d}^{t'}_{ik} + \hat{d}^{t'}_{ik}\zeta^{t'}_{ik} - q^{t'}_{ik}\right) \le w^t_{ik} - z^t_{ik}\right\}\\
& \ge 1-\epsilon^t_{ik}, \forall P \in \mathcal{P}, t \in T,\ i \in N,\ k \in K,\\
& \text{constraints } (4)-(10).
\end{aligned} \tag{12}$$

The model (12) is a distributionally robust optimization model with binary variables, and it is severely intractable. In the next section, we will replace the ambiguous chance constraint with its computationally tractable safe approximation.

4. Safe approximation of pre-positioning model

The aim of this section is to solve the probabilistic optimization problem (12) with an efficient method. A natural method is to replace the intractable constraint (11) with its safe approximation. In model (13), $\mathcal{S}$ is a system of computationally tractable convex constraints on variables (y, q, x, z, w) and additional variables u. Then, we can replace the intractable problem (12) with its computationally tractable approximation $\mathcal{S}$.

$$\min_{y,q,x,z,w,u}\left\{f: \begin{array}{c}(y,q,x,z,w,u) \text{ satisfies } \mathcal{S}\\ \\ \text{constraints}(4)-(10)\end{array}\right\}, \tag{13}$$

where the (y, q, x, z, w) component of every feasible solution of $\mathcal{S}$ is feasible for the chance constraint (11).

For the sake of simplicity, we consider the following scalar chance constraint inequality in the equivalent form:

$$P(\varphi) := \Pr_{\zeta^{t'}_{ik}\sim P}\left\{\zeta : \varphi^t_{ik} + \sum_{t'=0}^{t-1}\zeta^{t'}_{ik}\psi^{t'}_{ik} > 0\right\} \le \epsilon^t_{ik}, \quad \forall P \in \mathcal{P}, t \in T,\ i \in N,\ k \in K, \tag{14}$$

where

$$\varphi^t_{ik} = \sum_{t'=0}^{t-1}\left(\sum_{(i,j)\in A} x^{t'}_{ijk} - \sum_{(j,i)\in A} x^{t'}_{jik} + \bar{d}^{t'}_{ik} - q^{t'}_{ik}\right) - (w^t_{ik} - z^t_{ik}),$$

$$\psi^{t'}_{ik} = \hat{d}^{t'}_{ik}.$$

Consider the case of chance constraint (14), where the uncertainty $\zeta^{t'}_{ik}$ is a random variable with a probability distribution P known to belong to a given family $\mathcal{P}$. The family $\mathcal{P}$ is comprised of all distributions satisfying independent random perturbations with zero mean and subjected to a symmetric probability distribution. Under these assumptions, for all $t \in T, i \in N, k \in K$, consider the body of probability $P(\varphi)$ as

$$\varphi^t_{ik} + \sum_{t'=0}^{t-1}\zeta^{t'}_{ik}\psi^{t'}_{ik} > 0. \tag{15}$$

We assume that the uncertain data $\zeta^{t'}_{ik}$ resides in a given convex compact perturbation set Z^t_{ik} [32]. Consider the problem of bounding from above the probability $P(\varphi)$ of the event $\varphi^t_{ik} + \sum_{t'=0}^{t-1}\zeta^{t'}_{ik}\psi^{t'}_{ik} > 0$. Let us set $\Phi(\alpha^t_{ik}[\psi^0_{ik}; ...; \psi^{t-1}_{ik}]) =$

$(\alpha_{ik}^t)^2 \sum_{t'=0}^{t-1} \frac{1}{2}(\psi_{ik}^{t'})^2$. This should be a convex function such that $\mathrm{E}\{\exp\{\alpha_{ik}^t[\varphi_{ik}^t + \sum_{t'=0}^{t-1} \zeta_{ik}^{t'} \psi_{ik}^{t'}]\}\} \le \exp\{\alpha_{ik}^t \varphi_{ik}^t + \Phi(\alpha_{ik}^t[\psi_{ik}^0; \ldots; \psi_{ik}^{t-1}])\}$. The next step, we have $P(\varphi) \le \mathrm{E}\{\exp\{\alpha_{ik}^t[\varphi_{ik}^t + \sum_{t'=0}^{t-1} \zeta_{ik}^{t'} \psi_{ik}^{t'}]\}\} \le \exp\{\alpha_{ik}^t \varphi_{ik}^t + \Phi(\alpha_{ik}^t[\psi_{ik}^0; \ldots; \psi_{ik}^{t-1}])\} \le \epsilon_{ik}^t$. Thus, for a given ϵ_{ik}^t, and there exists $\alpha_{ik}^t > 0$ such that $\alpha_{ik}^t \varphi_{ik}^t + \Phi(\alpha_{ik}^t[\psi_{ik}^0; \ldots; \psi_{ik}^{t-1}]) \le \ln(\epsilon_{ik}^t)$, which is a safe approximation of the chance constraint (14). And the set $D_{ik}^t = \{\varphi_{ik}^t : \exists \alpha_{ik}^t > 0 : f(\alpha_{ik}^t) \equiv \alpha_{ik}^t \varphi_{ik}^t + \Phi(\alpha_{ik}^t[\psi_{ik}^0; \ldots; \psi_{ik}^{t-1}]) \le \ln(\epsilon_{ik}^t),\ t \in T,\ i \in N,\ k \in K\}$ is contained in the feasible set of the chance constraint (14). The next step, we give another safe approximation to reduce the conservatism of the above approximation. When $\varphi_{ik}^t \ge 0$, the function $f(\alpha_{ik}^t)$ attains its minimum on 0, i.e., $f(0) = 0$. When $\varphi_{ik}^t < 0$, the function $f(\alpha_{ik}^t)$ attains its minimum on $\frac{-(\varphi_{ik}^t)^2}{2\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2}$, i.e., $f(\frac{-\varphi_{ik}^t}{\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2}) = \frac{-(\varphi_{ik}^t)^2}{2\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2}$. In sum, when $\alpha_{ik}^t = 0$ or $\alpha_{ik}^t = \frac{-(\varphi_{ik}^t)^2}{2\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2}$, we arrive at the lower bound on above approximation, i.e., $\inf_{\alpha_{ik}^t > 0}\{f(\alpha_{ik}^t)\} \le \ln(\epsilon_{ik}^t)$. Since $\ln(\epsilon_{ik}^t) < 0$, the corresponding safe tractable approximation of (14) is given by the following inequality:

$$\varphi_{ik}^t + \sqrt{2\ln(1/\epsilon_{ik}^t)} \sqrt{\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2} \le 0, \forall t \in T,\ i \in N,\ k \in K. \tag{16}$$

Then, replacing the chance constraint in (14) with its safe tractable approximations rendered above, the original probabilistic violation is satisfied. When we set $\Omega_{ik}^t = \sqrt{2\ln(1/\epsilon_{ik}^t)}$, we arrive at another safe tractable approximation

$$\varphi_{ik}^t + \Omega_{ik}^t \sqrt{\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2} \le 0, \forall t \in T,\ i \in N,\ k \in K \tag{17}$$

of the chance constraint (14). With properly defined Ω_{ik}^t, every feasible solution of (17) is feasible for the chance constraint (14). This inequality is simply the robust counterpart form of inequality (15), with the ball perturbation set $Z_{ik}^{t\,Ball} = \{\zeta : \sqrt{\sum_{t'=0}^{t-1} (\zeta_{ik}^{t'})^2} \le \Omega_{ik}^t\}$.

Based on the above observation, we arrive at the following proposition:

Proposition 1. *By adopting the parameter that satisfies the family $\mathcal{P}$ comprised of zero mean probability distributions, constraint (17) is the robust counterpart of inequality (15) under perturbation set $Z_{ik}^{t\,Ball}$, and is a computationally tractable safe approximation of chance constraint (14).*

Proof. See Appendix A.1 for the proof. □

Consider the case that ignores the stochastic nature information of uncertain data and assumes that $\zeta_{ik}^{t'}$ just vary in $Z_{ik}^{t'\,Box} = [-\theta_{ik}^{t'}, \theta_{ik}^{t'}]$. The box safe approximation of the chance constraint (14) is

$$\varphi_{ik}^t + \sum_{t'=0}^{t-1} \theta_{ik}^{t'} |\psi_{ik}^{t'}| \le 0. \tag{18}$$

The box robust counterpart mentioned above guarantees "100% immunization against perturbations," which means that every robust solution is feasible for any realization of uncertainty (i.e., the randomly perturbed inequality in question with probability 1). The conservatism comparison of the above two safe approximations is determined by the dimension t of the uncertain parameter space. When $\Omega_{ik}^t \ge \theta_{ik}^{t'}\sqrt{t}$ $(\forall t' \in \{0, \ldots, t-1\})$, the diameter of perturbation set $Z_{ik}^{t\,Ball}$ is larger than that of perturbation set $Z_{ik}^{t'\,Box}$. In contrast, box one is more conservative. To obtain a less conservative robust optimization approximation with the same degree of constraint violation, we consider the following perturbation sets generated by combining the ball or polyhedron perturbation set with the box perturbation set.

First, we consider the perturbation set that is the intersection of the box and the ball:

$$Z_{ik}^{t\,BoxBall} = \left\{ \zeta \,\middle|\, |\zeta_{ik}^{t'}| \le \theta_{ik}^{t'}, \sqrt{\sum_{t'=0}^{t-1} (\zeta_{ik}^{t'})^2} \le \Omega_{ik}^t \right\}, \tag{19}$$

where $\theta_{ik}^{t'}$ and Ω_{ik}^t are the adjustable parameters controlling the size of the perturbation set.

Since the approximation of the robust counterpart type is a semi-infinite programming problem, in order to reformulate (15) as a tractable optimization problem, we often use the theorem of duality to eliminate the left hand maximization and incorporate the dual formulation into the original constraint.

Theorem 1. *Given the defined demand perturbation set $Z^{BoxBall}$, then the corresponding box-ball robust counterpart of inequality (15) is given by the following conic quadratic constraint:*

$$\begin{cases} \varphi_{ik}^{t} + \left[\sum_{t'=0}^{t-1} \theta_{ik}^{t'} u_{ik}^{t'} + \Omega_{ik}^{t} \sqrt{\sum_{t'=0}^{t-1} (\eta 1_{ik}^{t'})^2} \right] \leq 0, \quad \forall t \in T,\ i \in N,\ k \in K, \\ -u_{ik}^{t'} \leq \psi_{ik}^{t'} - \eta 1_{ik}^{t'} \leq u_{ik}^{t'}, \qquad \forall t \in T,\ t' \in \{0,\ldots,t-1\},\ i \in N,\ k \in K, \end{cases} \tag{20}$$

where $u_{ik}^{t'}$ and $\eta 1_{ik}^{t'}$ represent the dual variables.

Proof. See Appendix A.2 for the proof. □

With the different values of $\theta_{ik}^{t'}$ and Ω_{ik}^{t}, the coincidence state between the box and the ball also changes. When $\Omega_{ik}^{t} \leq \theta_{ik}^{t'}$ ($\forall t' \in \{0,\ldots,t-1\}$), the ball is embedded into the box. The uncertain space is determined by the ball perturbation set. When $\theta_{ik}^{t'} \leq \Omega_{ik}^{t} \leq \theta_{ik}^{t'}\sqrt{t}$ ($\forall t' \in \{0,\ldots,t-1\}$), the box and ball intersect, where t is the dimension of the uncertain parameter $\zeta_{ik}^{t'}$. The uncertain space is determined by the two perturbation sets. When $\theta_{ik}^{t'}\sqrt{t} \leq \Omega_{ik}^{t}$ ($\forall t' \in \{0,\ldots,t-1\}$), the box is ringed by the ball. The uncertain space is determined by the box perturbation set, which is the same as (18).

Second, we can further extend the box-ball perturbation set to the larger and simpler set, that is, the box-polyhedral perturbation set,

$$Z_{ik}^{t\,Boxpolyhedral} = \left\{ \zeta \,\middle|\, |\zeta_{ik}^{t'}| \leq \theta_{ik}^{t'}, \sum_{t'=0}^{t-1} |\zeta_{ik}^{t'}| \leq \Gamma_{ik}^{t} \right\}, \tag{21}$$

where $\theta_{ik}^{t'}$ and Γ_{ik}^{t} are the adjustable parameters controlling the size of the perturbation set.

We also use the theorem of duality to reformulate (15) into a tractable optimization form.

Theorem 2. *Given the defined demand perturbation set $Z^{BoxPolyhedral}$, then the corresponding box-polyhedral robust counterpart of inequality (15) is equivalent to the following constraint:*

$$\begin{cases} \varphi_{ik}^{t} + \left[\sum_{t'=0}^{t-1} \theta_{ik}^{t'} \eta 1_{ik}^{t'} + \Gamma_{ik}^{t} u_{ik}^{t} \right] \leq 0, \quad \forall t \in T,\ i \in N,\ k \in K, \\ \eta 1_{ik}^{t'} + u_{ik}^{t} \geq \psi_{ik}^{t'}, \qquad \forall t \in T,\ t' \in \{0,\ldots,t-1\},\ i \in N,\ k \in K, \\ \eta 1_{ik}^{t'}, u_{ik}^{t} \geq 0, \qquad \forall t \in T,\ t' \in \{0,\ldots,t-1\},\ i \in N,\ k \in K, \end{cases} \tag{22}$$

where $\eta 1_{ik}^{t'}$ and u_{ik}^{t} represent the dual variables.

Proof. See Appendix A.3 for the proof. □

Similarly, with the different values of $\theta_{ik}^{t'}$ and Γ_{ik}^{t}, the coincidence state between the box and the polyhedron also changes. When $\Gamma_{ik}^{t} \leq \theta_{ik}^{t'}$ ($\forall t' \in \{0,\ldots,t-1\}$), the polyhedron is embedded into the box. The uncertain space is determined by the polyhedral perturbation set. When $\theta_{ik}^{t'} \leq \Gamma_{ik}^{t} \leq \theta_{ik}^{t'} t$ ($\forall t' \in \{0,\ldots,t-1\}$), there is an intersection between the box and the polyhedron, where t is the dimension of the uncertain parameter $\zeta_{ik}^{t'}$. The uncertain space is determined by the two perturbation sets. When $\theta_{ik}^{t'} t \leq \Gamma_{ik}^{t}$ ($\forall t' \in \{0,\ldots,t-1\}$), the box is embraced by the polyhedron. The uncertain space is determined by the box perturbation set, which is the same as (18).

Next, based on the box-ball and box-polyhedral perturbation sets, we arrive at the following results.

Theorem 3. *In the case of constraint (20), if $\{\zeta_{ik}^{t'}\}_{t' \in \{0,\ldots,t-1\}}$ are independent and subject to a symmetric probability distribution, then for every $\theta_{ik}^{t}, \Omega_{ik}^{t} \geq 0$, the following probability of constraint violation holds that*

$$Pr\left\{ \varphi_{ik}^{t} + \sum_{t'=0}^{t-1} \zeta_{ik}^{t'} \psi_{ik}^{t'} > 0 \right\} \leq \exp\{-\Omega_{ik}^{t\,2}/(2(\theta_{ik}^{t})^2)\}, \forall t \in T,\ i \in N,\ k \in K, \tag{23}$$

where $\exp\{-\Omega_{ik}^{t\,2}/(2(\theta_{ik}^{t})^2)\}$ represents the degree of the constraint violation, i.e., the probability that the constraint is violated. Every feasible solution to these constraints is feasible for the chance constraint inequality (14) with probability of at least $1 - \exp\{-\Omega_{ik}^{t\,2}/(2(\theta_{ik}^{t})^2)\}$.

Proof. See Appendix A.4 for the proof. □

In addition, to satisfy the constraint with probability at least $1 - \epsilon_{ik}^{t}$, we denote $\exp\{-\Omega_{ik}^{t\,2}/(2(\theta_{ik}^{t})^2)\} \leq \epsilon_{ik}^{t}$. Then, we can obtain the safe parameter with $\Omega_{ik}^{t} \geq \theta_{ik}^{t}\sqrt{2\ln(1/\epsilon_{ik}^{t})}$.

For the box+polyhedral uncertainty set, we obtain a similar theorem.

Fig. 1. The map of Circum-Bohai Sea Region since 1968.

Theorem 4. *In the case of constraint (22), if $\{\zeta_{ik}^{t'}\}_{t'\in\{0,\ldots,t-1\}}$ are independent and subject to a symmetric probability distribution, then for every $\theta_{ik}^t, \Gamma_{ik}^t \geq 0$, the following probability of constraint violation holds that*

$$Pr\left\{\varphi_{ik}^t + \sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\psi_{ik}^{t'} > 0\right\} \leq \exp\{-\Gamma_{ik}^{t\,2}/(2t(\theta_{ik}^t)^2)\}, \forall t \in T,\ \ i \in N,\ \ k \in K, \tag{24}$$

where $\exp\{-\Gamma_{ik}^{t\,2}/(2t(\theta_{ik}^t)^2)\}$ represents the degree of constraint violation, i.e., the probability that the constraint is violated. Every feasible solution to these constraints is feasible for the chance constraint inequality (14) with probability of at least $1 - \exp\{-\Gamma_{ik}^{t\,2}/(2t(\theta_{ik}^t)^2)\}$.

Proof. See Appendix A.5 for the proof. □

In addition, to satisfy the constraint with probability of at least $1-\epsilon_{ik}^t$, we denote $\exp\{-\Gamma_{ik}^{t\,2}/(2t(\theta_{ik}^t)^2)\} \leq \epsilon_{ik}^t$. Then, we can obtain the safe parameter with $\Gamma_{ik}^t \geq \theta_{ik}^t\sqrt{2t\ln(1/\epsilon_{ik}^t)}$. The quantity $\Gamma_{ik}^t/\sqrt{t}$ plays the same role as the quantity Ω_{ik}^t.

The optimal object value of the robust approximation problem should be as close to the real value of the chance constraint problem as possible. This means that the employed perturbation set should be as small as possible while ensuring that every feasible solution is feasible for the chance constraint. However, a smaller perturbation set may bring more complex, and even computationally intractable problems. Thus, a suitable set should take into account computational complexity and conservatism. In the next section, we will provide a case study to verify the performance of different perturbation sets.

5. Case study

The pre-positioning emergency supply problem can effectively reduce the negative effects of natural disasters. In this section, we focus on the pre-positioning emergency supply problem prepared for earthquake in Circum-Bohai Sea Region of China. The solution procedure is solved using CPLEX 12.7.1 optimization software, and all the experiments are performed on an INTEL Core 4 CPU with a 3.4 gigahertz processor and 24 gigabytes of RAM.

5.1. Problem description

The Circum-Bohai Sea Region has an important strategic position in China, as it includes the political, economic and cultural center of the country. The region is located near the North China seismic belt, which includes 157 cities with a population of approximately 260 million people and with a size of approximately 1120 thousand square kilometers. According to the national earthquake emergency response plan of China, the corresponding emergency response plan should be activated for earthquakes with magnitudes greater than 4.0. Fig. 1 presents the Circum-Bohai Sea Region's seismic map from

Fig. 2. Pre-positioning network of the Circum-Bohai Sea Region.

Table 1
Sizes, fixed costs, and storage capacity of facilities.

Size	$F_l(CNY)$	$\gamma_l(m^3)$
Small	11,205,000	10,458
Medium	18,649,800	20,084
Large	49,470,000	61,110

the United States Geological Survey (USGS), and the dots depict all earthquakes with greater than 4.0 magnitude since 1968. This region has suffered from frequent disasters and is one of the main earthquake monitoring areas for the China National Commission for Disaster Reduction.

Thirty candidate cities are selected with convenient transportation in the region to host the facilities. The pre-positioning network is presented in Fig. 2. Based on the construction standard for a relief supplies reserve base $121-2009$, we choose three different types of facilities, i.e., large, medium and small facilities. The capacity of the facility equals the effective area of the facility multiplied by its height (4.2m). Usually, the larger the facility is, the lower the construction cost per square meter. Based on the project cost analysis, the estimated construction costs of the three types of facilities are 3400, 3900 and 4500 CNY per square meter, respectively. The relevant parameters of the facilities are shown in Table 1. The facilities should be located adjacent to rail terminals or highway entrances so that after an earthquake, the emergency commodities can quickly meet the needs of the affected people.

Table 2
Sizes, fixed costs, and storage capacity of facilities.

Arcs	*Arc capacity(ton)*	*Arcs*	*Arc capacity(ton)*	*Arcs*	*Arc capacity(ton)*
1 – 2	170	10 – 11	460	18 – 19	410
1 – 3	390	10 – 12	350	19 – 20	30
1 – 5	90	10 – 14	960	20 – 21	390
1 – 6	710	11 – 13	970	20 – 22	80
2 – 6	50	12 – 13	250	21 – 22	100
2 – 7	250	12 – 14	20	22 – 23	480
3 – 4	360	12 – 15	150	22 – 26	990
4 – 7	110	13 – 14	320	23 – 28	120
5 – 6	40	13 – 15	630	24 – 25	30
5 – 9	10	14 – 16	1310	25 – 26	10
6 – 8	470	15 – 17	540	26 – 27	920
6 – 9	10	15 – 24	20	27 – 28	650
8 – 11	790	16 – 17	290	27 – 29	270
9 – 10	70	16 – 18	880	27 – 30	140
9 – 11	20	17 – 22	1130	29 – 30	240

Several assumptions are made as follows. First, the candidate cities are chosen by considering convenient railway transportation.[2] Railway transportation has the advantages of high transportation capacity, low price and high speed. The bearing capacity of an arc in one day, as shown in Table 2, is determined by the number of railway lines between cities. In this paper, the emergency commodities are supplied from pre-positioning facilities opened in these candidate cities to the affected areas via the railway. Second, due to the high anti-seismic capabilities, we assume that the facilities and the roads linking them will not be disrupted if the earthquake does not exceed 7.0 Ms. Third, we consider five basic emergency commodities: food, water, medical kit, shelter and clothing. Last, the response time of the pre-positioning network after an earthquake is set to within 30 days. We assume that demands are only generated in the first 20 days of the planning horizon. In practice, however, due to the road capacity constraint, the emergency commodities will be delayed to transported to the affected area. Thus, we set up an extra 10 days to replenish the first 20 days of unsatisfied demands. The commodities require one day to be transported to a city.

The demand for relief supplies is related to the number of affected people, which is driven by the magnitude of the earthquake. First, we need to approximately estimate the probability of an earthquake. We use historical data from the USGS on the numbers of earthquakes greater than 4.0 during the last 50 years, i.e., $H = \{\tau | 1968 \leq \tau \leq 2018\}$. Second, we take the urban area of the candidate point as the center and 50 km as the radius to obtain the magnitude and number of earthquakes. Because of the small number of earthquakes, it is difficult and impractical to exactly estimate the probability distribution based on historical data. Thus, we take the mean of the earthquake magnitude as the nominal value to estimate the disaster frequency, and the magnitude of the earthquake is a nearly symmetric distribution.

According to [49], the disaster frequency per year and the expected average earthquake magnitude can be roughly estimated as

$$fr_i = \frac{m_i}{50}, i \in N,$$
$$Ma_i^* = \frac{\sum_{\tau \in H} Ma_i^\tau}{m_i}, i \in N,$$

where m_i is the number of earthquakes at node i, and Ma_i^τ is the magnitude of an earthquake occurring at node i time τ.

Since probabilistic seismic hazard assessment is time-independent, by incorporating the probability changes caused by large mainshocks [50], the probability that the number m_i of earthquakes with magnitude Ma_i greater than Ma_0 at the chosen point within T time units is given according to a Poisson distribution.

$$\Pr_i(Ma_i \geq Ma_0) = \frac{(\varphi(Ma_i) \cdot T)^{m_i}}{m_i!} \cdot e^{(-\varphi(Ma_i) \cdot T)}, i \in N,$$

where $\varphi(Ma_i)$ is the mean frequency of disasters with magnitudes greater than Ma_0 in one time unit. In addition, the mean frequency of earthquakes $\varphi(Ma_i)$ is an empirical recurrence function given by

$$\varphi(Ma_i) = fr_i \cdot e^{-\frac{Ma_i}{Ma_i^*}}, i \in N.$$

[2] We omit sea shipping and road transportation in this case. First, in this region, sea shipping has only one potential point link, i.e., Weihai (node 30) and Dalian (node 7). Further, sea shipping is full of uncertain risk, especially in bad weather. In general, earthquakes may bring tidal waves that hit coastal areas. For the port cities in our problem (Weihai and Dalian), sea shipping is not safe. Second, although road transportation has advantages in terms of the flexibility of both the location and the time of delivery, it is appropriate for short distances and "last mile" transportation assignments, which are microscopic scale decisions. In the Circum-Bohai Sea Region case, the decision makers are focused on the macroscopic scale decisions, for which rail is more suitable.

Table 3
Number and average magnitude of historical earthquakes .

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number (m_i)	1	28	1	0	0	6	1	3	0	2	64	1	5	0	1
Average magnitude (Ma_i^*)	4.8	4.8	4.5	0	0	4.7	4.6	4.4	0	4.3	4.8	4.6	5	0	5
Node	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Number (m_i)	15	0	2	0	0	6	0	1	0	2	0	0	0	2	1
Average magnitude (Ma_i^*)	4.3	0	4.9	0	0	4.7	0	1	0	4.4	0	0	0	4.1	4.0

Table 4
Nominal demands at 30 candidate cities (Unit, 0-19 days).

Commodity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Water	42	325	7	0	0	37	34	42	0	192	2177	23	417	0	40
Food kit	28	217	4	0	0	25	22	28	0	128	1451	15	278	0	27
Medical kit	284	2170	47	0	0	250	226	284	0	1285	14514	153	2780	0	272
Shelter	177	1356	29	0	0	156	141	177	0	803	9071	96	1737	0	170
Clothing	142	1085	23	0	0	125	113	142	0	642	7257	76	1390	0	136
Commodity	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Water	47	0	83	0	0	258	0	0	0	19	0	0	0	59	11
Food kit	31	0	55	0	0	172	0	0	0	13	0	0	0	39	7
Medical kit	319	0	553	0	0	1726	0	0	0	130	0	0	0	392	75
Shelter	199	0	346	0	0	1079	0	0	0	81	0	0	0	245	647
Clothing	159	0	276	0	0	863	0	0	0	65	0	0	0	196	637

Therefore, we calculate the probability ω_i that more than one earthquake will occur within one year at node i by

$$\omega_i = 1 - e^{-\varphi(Ma_i)}, i \in N.$$

In this paper, we consider earthquakes greater than magnitude 6.0 that can cause serious losses. The relevant historical earthquake data with $Ma \geq 4$ in recent 50 years are shown in Table 3.

The minimum standards of emergency commodities for humanitarian response are derived from [51] and Construction Standard 121 – 2009. More specifically, using the compressed biscuits as emergency food, each person receives two pieces per day to meet 2100 kcal energy requirements. Food is served in the food kit and 1000 pieces is considered to be a unit kit. Water is assumed to be in units of 1000 liters, and a person needs three liters of water daily. A single medical kit is designed to serve 50 persons per day. Food, water and medical kits as periodical emergency commodities should meet the daily needs at each candidate city for the entire emergency period. A 12 square meter shelter dedicated to disaster relief can accommodate 4 persons. Each person should be provided 2 sets of emergency clothing. Considering non-periodical emergency commodities, shelter and clothing should be transported to each candidate city in the initial time periods [52]. We calculate the total demand for non-periodical emergency commodities to be determined within 20 days and to be distributed the same amount each day for simplicity. Nevertheless, our results can be easily extended to distribute the commodities in any proportion.

Demand at each candidate city is estimated on the basis of the population multiplied by the proportion of assistance needed and the daily demand for emergency commodities. The assistance proportion is determined by the number of people affected by the earthquake. Generally speaking, earthquakes greater than magnitude 6.0 can cause MSK intensity VIII, which can cause more than 30% of buildings to collapse [53].

Based on the above description, the demand for each candidate city can be approximately calculated by the following formulation:

$$d_{ik}^t = d_k^t \cdot n_i \cdot \omega_i \cdot 30\%, \tag{25}$$

where n_i denotes the population of candidate city i, and d_k^t represents the daily demand of emergency commodity k.

Table 4 shows the demands at 30 candidate cities in detail. In Table 4, the demands in the first 20 days are the deterministic value, i.e., the nominal value.[3] The deterministic model assumes deterministic demands equal to the values shown in Table 4. However, in the real world, demand can fluctuate unexpectedly because it has many sources, including the estimated error in affected people, damaged emergency commodity and mutual help between different areas. The upper and lower bounds for the earthquake magnitude is approximately 10% of the nominal value. Thus, the positive constant perturbation $\hat{d}_{ik}^t$ is estimated to be 10% of the nominal demand.

Table 5
Unit acquisition price, volume, weight and transportation costs for commodities.

Commodity	$a_k(CNY/unit)$	$v_k(m^3/unit)$	$b_k(ton/unit)$	$c_k(CNY/unit \cdot km)$
Water	1000	1	1	0.28
Food kit	6000	0.059	0.25	0.017
Medical kit	800	0.05	0.015	0.014
Shelter	1200	0.312	0.052	0.087
Clothing	380	0.158	0.014	0.044

Table 6
Results of deterministic and two robust counterpart' models under one-fold penalty factor.

Model	*Objective*	*Facility size*			*Commodity(Unit)*[8]				
	(CNY)	*Large*	*Medium*	*Small*	*W*	*F*	*M*	*S*	*C*
Deterministic	1,533,147,507	11,13	2,8,10,15	1,18,21,25,29	76,260	50,800	509,200	318,200	254,540
Box − polyhedral	1,635,588,889	10,11,13	2,21	1,8,18,25,29	81,463	54,260	543,889	339,877	271,894
Box − ball	1,635,229,311	10,11,13	2,21	1,8,18,25,29	81,448	54,256	543,845	339,849	271,858

* W=Water,F=Food kits,M=Medical kits,S=Shelter,C=Clothing.

Table 5 lists the parameters for the acquisition cost a, the volume v, the weight b and the transportation cost c per unit distance. More specifically, the acquisition cost is a rough estimation of the current market price; the volume and weight depend on the actual specifications of the commodities; and the transportation cost is estimated according to the China railway freight network 95306. In addition, the holding cost h is assumed to be 10% of the acquisition cost per year [8]. This cost means that if the commodities are not all used by the end of the disaster, a management cost is incurred and a salvage value exists, e.g., if the holding cost equals 0; this represents the supply-demand balance of commodities. Because transportation delay is a major impediment to commodity distribution, the unmet demand one-fold penalty factor is estimated to be 1 to 8 times the purchase price, which increases cubically along the time horizon. The holding and penalty costs are useful for illustration purposes but values may not match between different locations or different levels of emergency.

In Section 4, we introduce 3 perturbation sets to describe uncertain demands. To control the size of the generated intervals of uncertainty, we define the adjustable parameters of the box perturbation set $\theta_{ik}^{t'} = 1,$ i.e., the perturbation variable $\zeta_{ik}^{t'}$ varies in $[-1, 1]$. The box robust counterpart is feasible for the randomly perturbed inequality with probability 1. However, the perturbation set is not necessarily defined to guarantee 100% protection against perturbations. For ease of calculation, we set the risk degree ϵ_{ik}^{t} to be the same value for all $t \in T, i \in N, k \in K$. Therefore, for box-ball and box-polyhedral perturbation sets, we assume that the robust optimal solution satisfies the uncertainty-affected constraint with probability of at least $1 - \epsilon_{ik}^{t} = 0.99$. According to Theorems 1 and 2, the adjustable parameter $\Omega_{ik}^{t} = \sqrt{2\ln(1/\epsilon_{ik}^{t})} = 3.035$ and $\Gamma_{ik}^{t} = \sqrt{2\ln(1/\epsilon_{ik}^{t})}\sqrt{20} = 13.572,$ which guarantee the same probability of constraint violation.

5.2. Results analysis

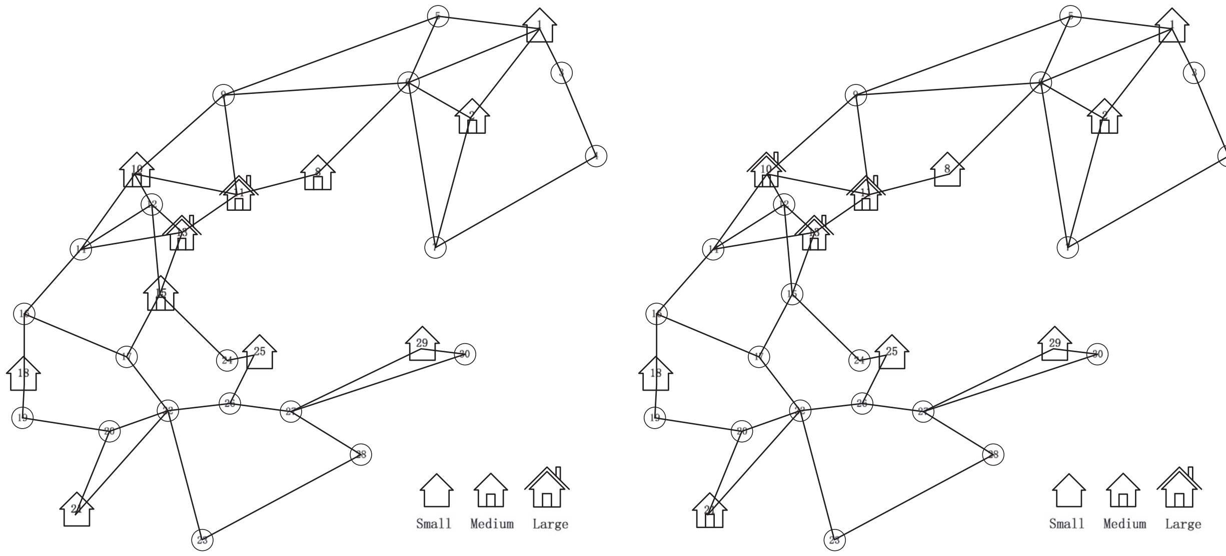
In this section, to validate the distributionally robust optimization approach and determine the value of considering uncertainty, we evaluate computational results and analyze the performance of the proposed distributionally robust emergency supplies pre-positioning model. We will make a comparative discussion from the following three aspects: distributionally robust counterpart's solutions VS. deterministic solutions, distributionally robust counterpart's solutions VS. robust counterpart's solutions and multi-period model's solutions VS. single-period model's solutions.

5.2.1. Distributionally robust VS. deterministic

In this case, we compare solutions between the distributionally robust model and its deterministic counterpart to evaluate their performance in addressing demand uncertainty. Table 6 shows the results from the deterministic model and the two distributionally robust counterpart (RC) models under one-fold penalty factor, including the "box-polyhedral" based model and the "box-ball" based model. In this tabulation, the entries in the column *Objective* show the optimal cost. The entries in the column *Facility size* represent the number of facilities of different sizes, and *Commodity* represents the total pre-positioning quantity of different types of commodities.

In Tables 6, 7 pre-positioning facilities are opened in the network of the deterministic model, with 2 large ones, 3 medium ones and 2 small ones. Similarly, 9 pre-positioning facilities are opened in the network of box-polyhedral and box-ball RC models, with 3 large ones, 2 medium ones and 4 small ones. The numbers and locations of the pre-positioning

[3] In the nominal model, the perturbation value of demand in constraint (3) is zero, that is to say, demand is a deterministic value, not a random variable. In this paper, the nominal model is the same as the deterministic model.



(a) Deterministic (b) Box-ball and Box-polyhedral

Fig. 3. Facility pre-positioning locations from the deterministic, box-polyhedral RC and box-ball RC models.

Table 7
List of deterministic and two robust counterpart' costs under one-fold penalty factor.

Cost list	*Deterministic(CNY)*	*Box − polyhedral (CNY)*	*Box − ball (CNY)*
Fixed cost	229,564,200	24,1734,600	24,1734,600
Acquisition cost	1,266,985,200	1,353,307,826	1,353,186,160
Transportation cost	5,671,286	5,055,850	5,048,059
Inventory cost	28,283,681	31,840,351	31,610,228
Penalty cost	2,643,140	3,650,262	3,650,264

facilities in the three models are shown in Fig. 3. The objective function value of the deterministic model is approximately 1533.15 million CNY, including 229.56 million CNY in fixed costs, 1266.99 million CNY in acquisition costs, 5.67 million CNY in transportation costs, 28.28 million CNY in inventory costs, and 2.64 million CNY in penalty costs for unmet demand. The objective function values of the box-polyhedral and box-ball RC models are increased to 1635.59 million CNY and 1635.23 million CNY, respectively. Changes in the specific costs for the box-polyhedral and box-ball RC models under one-fold penalty factor are listed in Table 7.

From the comparison of the above results, we find that the robust feasible solution follows the "worst-case-oriented" criterion. Therefore, it is reasonable for the distributionally robust counterpart model to have higher costs when faced with uncertain demand. The two distributionally robust counterpart models have very similar solutions. The difference is that the solutions of the box-ball RC are slightly better than those of the box-polyhedral RC. However, the box-polyhedral RC can be represented by a system of linear constraints, which can be easily addressed by commercial solvers. In contrast, the box-ball RC is an example of conic quadratic programming, which may lead to a hard or even intractable computation. We find that the box-polyhedral RC solution yields high-quality solutions with quite short CPU times. Next, we will take the box-polyhedral RC model as an example to illustrate the performance of the distributionally robust optimization approach.

Due to the unpredictability of uncertain demand, the deterministic model is likely to obtain infeasible solutions. In response to the perturbation of demand, the distributionally robust model obtains a relatively conservative solution. However, the distributionally robust model can generate solutions that satisfy all constraints for any perturbations of uncertain demand.

In reality, the demand of the deterministic model cannot be absolutely reliable. The purpose of following experiment is to observe that what will we suffer when sticking to the nominal solution when the actual demand is against it. Therefore, we assume that the demands of commodities are realizations of random variables varied around the nominal demands. We assume that the demands of commodities are realizations of random variables varied around the nominal demands. According to the fluctuation of historical earthquake magnitude, we assume fluctuation value drifts in a 10% margin of the nominal demand. Moreover, we assume that the demands of the deterministic model take the nominal value and extreme value in the respective segments with probabilities 0.5 each.

Based on the results in Table 6, we analyze the expected values of the deterministic and box-polyhedral RC models under constraint violation. In the pre-positioning problem, a disaster will incur different risk degrees ϵ_{ik}^{t} with respect to different

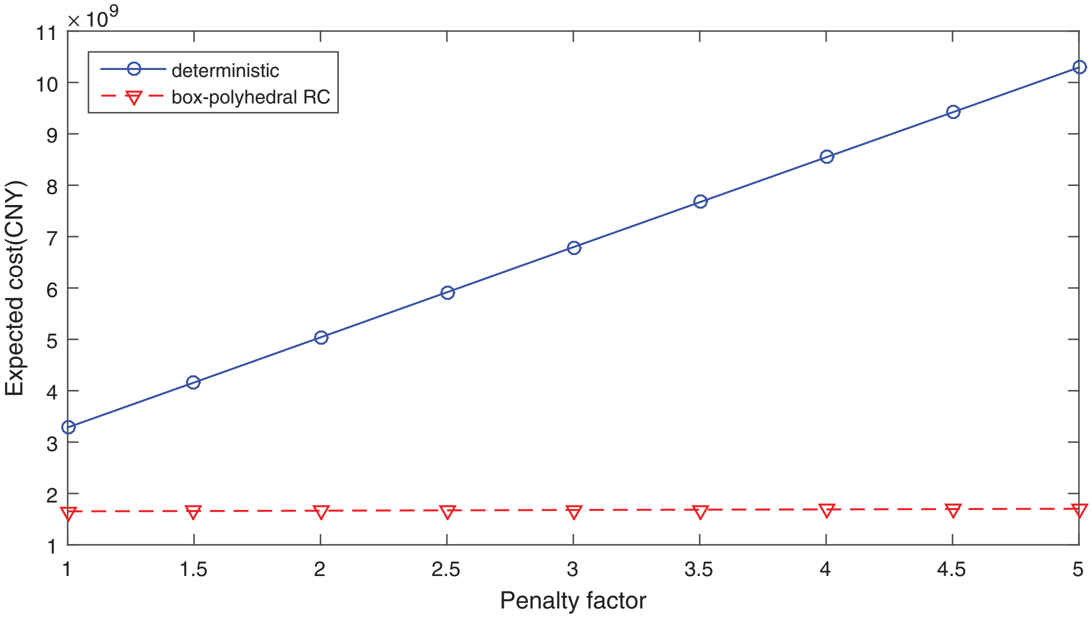


Fig. 4. Expected cost with different penalty factors.

cities, times and commodities. The stability of commodity supply plays a significant role in disaster prevention. To prevent underestimating risk, we take the maximum risk degree ϵ to calculate the expected costs of the deterministic model and the box-polyhedral RC model, as shown below,

$$E[cost] = f_{cs} * (1 - \epsilon) + f_{cv} * \epsilon, \tag{26}$$

where $\epsilon = \max_{t\in T, i\in N, k\in K} \epsilon_{ik}^t$, f_{cs} represents the optimal value of the deterministic model or the box-polyhedral RC model, and f_{cv} represents the value of the deterministic model or the box-polyhedral RC model when there is a constraint violation. That is to say, f_{cv} represents the penalty value when sticking to the optimal solution when the demand is against it. For the aim of simplicity, we assume that ϵ_{ik}^t are the same for all $t \in T, i \in N, k \in K$.

Based on the calculation of Eq. (26), the expected cost of the nominal model is 3284.79 million CNY. With the requires risk level $\epsilon = 1\%$, the expected cost of box-polyhedral RC is 1652.45 million CNY. In Table 6, we can see that although the total cost yielded by the box-polyhedral RC model is 6.68% more than that provided by the deterministic model, the expected cost of the box-polyhedral RC model is less than the 98.78% reduction in the actual expected cost. These results show that a small perturbation of demand may heavily affect the quality of the solution. The price of robustness may effectively be a reduction in the infeasibility of the optimal solution.

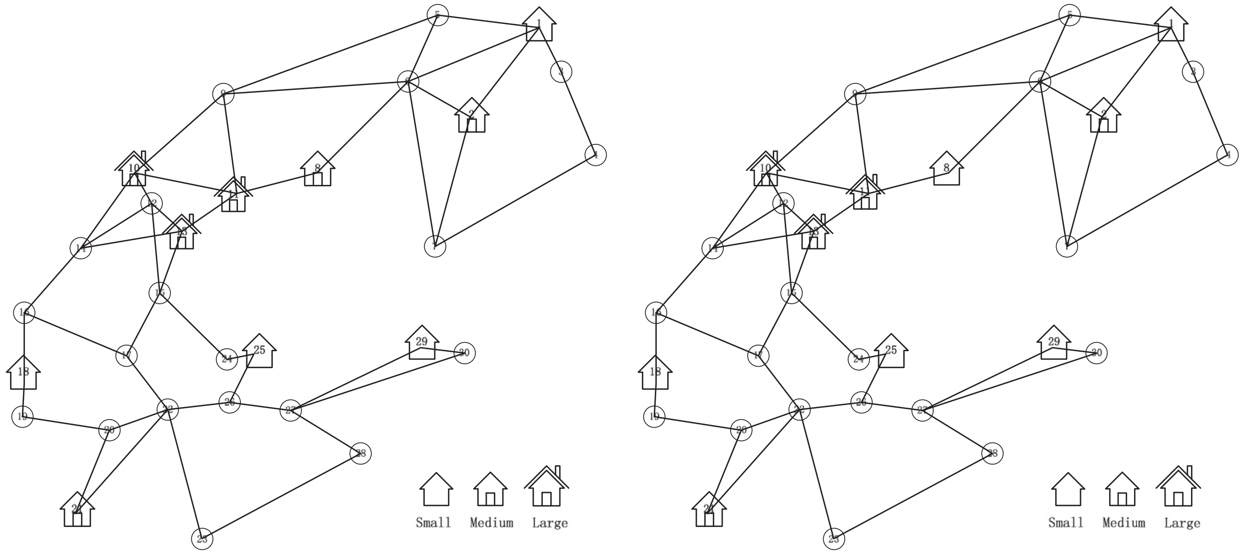
Fig. 4 shows the expected costs of the deterministic and box-polyhedral RC models with different penalty factors. When increasing the penalty factor, the expected costs of the box-polyhedral RC model see only minor changes, whereas the expected costs of the deterministic model show an increasing trend. When the penalty factor is small, large changes in decision making will not have a significant impact on the objective, and robustness may not be cost-effective. However, when the penalty factor is large, small fluctuations in decision making will have a large impact on the objective. For the deterministic model, the expected cost of the fivefold penalty factor is 213.35% more than that of the onefold penalty factor, whereas for the box-polyhedral RC model, the expected cost of the fivefold penalty factor is just 3.04% more than that of the onefold penalty factor. These results indicate that the distributionally robust optimization model can adapt to more urgent problems. Thus, when the penalty costs are subject to prediction or measurement errors, the distributionally robust model can minimize the total cost on the basis of ensuring a certain rate of demand satisfaction. Under the background of emergency supplies, it is better to set the penalty factor to a relatively high value. In the next section, we will compare the performance of the distributionally robust and robust models.

5.2.2. Distributionally robust vs. robust

In this section, we compare the solutions of the distributionally robust and robust models to evaluate their performance when dealing with demand uncertainty. The robust model uses (18) to replace the chance constraint (11). The robust model is the “worst-case-oriented” model, i.e., the box RC model. Typically, the perturbations of uncertain parameters are of a stochastic nature. Based on historical data, we usually have only partial distribution information for random perturbations. The solutions of the robust model (box RC) usually make the constraint valid with probability 1, which means it is too

Table 8
Statistics of box and box-polyhedral robust counterpart models under one-fold penalty factor.

	Facility size		Cost list(CNY)		Commodity(Unit)	
Box	*Large*	10,11,13	*Fixedcost*	249179400	*Water*	83,886
	Medium	2,8,21	*Acquisitioncost*	1393683720	*Foodkits*	55,880
	Small	1,18,25,29	*Transportationcost*	5007993	*Medicalkits*	560,120
			Inventorycost	30365095	*Shelter*	350,020
			Penaltycost	3650262	*Clothing*	279,994
Box − polyhedral	*Large*	10,11,13	*Fixedcost*	241734600	*Water*	81,463
	Medium	2,21	*Acquisitioncost*	1353307826	*Foodkits*	54,260
	Small	1,8,18	*Transportationcost*	5055850	*Medicalkits*	543,889
		25,29	*Inventorycost*	31840351	*Shelter*	339,877
			Penaltycost	3650262	*Clothing*	271,894



(a) Box (b) Box-polyhedral

Fig. 5. Facility pre-positioning locations of box and box-polyhedral RC models.

conservative. Next, we will analyze the distributed robust model and see that it can not only reduce the price of robustness but also keep the results stable with an increase of uncertainty level.

Table 8 presents the statistics for the box and box-polyhedral RC models under one-fold penalty factor, and the numbers and locations of the pre-positioning facilities are shown in Fig. 5. The box RC guarantees 100% immunization against perturbations, meaning that the expected cost is equal to the objective function value. Then, the expected cost from the box RC model is 1688.46 million CNY, which is 9.7% more than the deterministic cost. This solution is robust for any possible realization of uncertainty, and there is no constraint violation. However, the decision maker may prefer the less conservative model.

To reduce the degree of conservativeness, the decision maker can allow for a certain degree of constraint violation that may lead to a more optimal solution. As mentioned above, the box-polyhedral RC can employ a degree of constraint violation as low as 1%. In general, what size of constraint violation is necessary to ensure that the solution is the most optimal? To illustrate the answer to this question, we use the probability of constraint violation ϵ varied from 0 to 0.1 in intervals of 0.005. The expected cost from the box-polyhedral RC model can be calculated by Eq. (26). The distribution of results is depicted in Fig. 6.

Fig. 6 shows the trade-off relationship between expected cost and constraint violation probability. For the box RC model, the expected cost remains constant as the confidence level increases. This means that the box RC provides the most conservative solution and that every feasible solution to the box RC is feasible here. For the box-polyhedral RC, we can see that the expected cost is not always a monotonically increasing or decreasing function with respect to the risk. The expected cost of the box-polyhedral RC model is lower than that of the box RC model with confidence level $\epsilon = 0.05$. The expected cost of the box-polyhedral RC reaches the minimum cost of 1635.59 million CNY at the approximate risk $\epsilon = 0.009$. Note that when $\epsilon = 0$, the box and box-polyhedral RCs have the same solution.

Based on the above experiments, we test the performance of the box-polyhedral RC and box RC models under different risks. Two perturbation sets are available for constructing robust counterpart models. The solutions of the models have

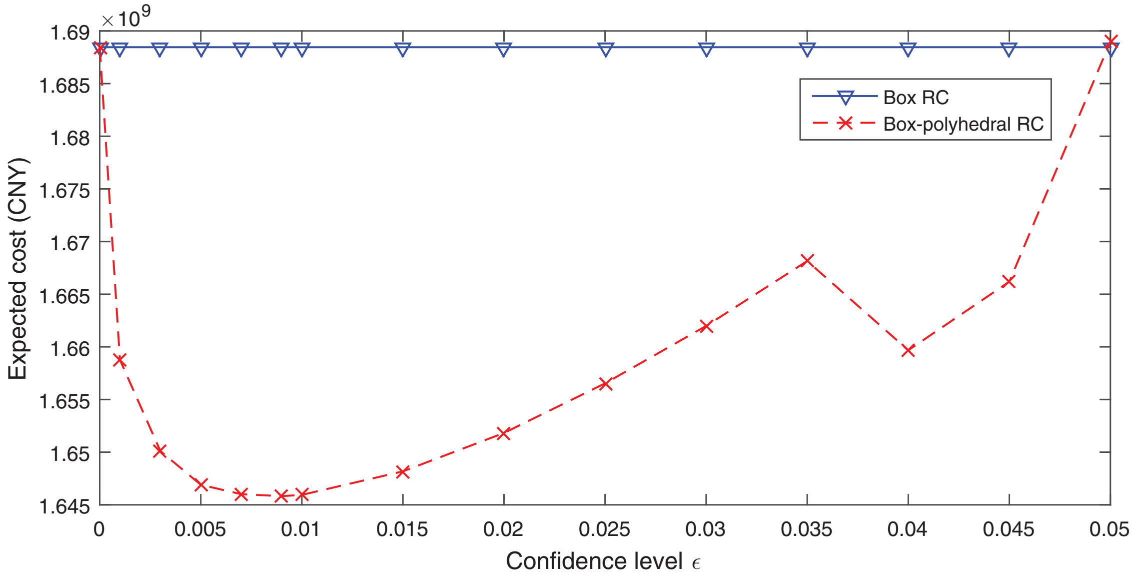


Fig. 6. Expected cost with constraint violation probability.

Table 9
Statistics of static and dynamic models under deterministic environment.

	Facility size		Cost list(CNY)		Commodity(Unit)	
Static	*Large*	6,10,11,13	*Fixedcost*	209085000	*Water*	76260
	Medium		*Acquisitioncost*	1284865200	*Foodkits*	50800
	Small	30	*Transportationcost*	10521679	*Medicalkits*	507850
			Inventorycost	0	*Shelter*	330200
			Penaltycost	0	*Clothing*	266540
Dynamic	*Large*	11,13	*Fixedcost*	229564200	*Water*	76260
	Medium	2,8,10,15	*Acquisitioncost*	1266985200	*Foodkits*	50800
	Small	1,18,21,25,29	*Transportationcost*	5671286	*Medicalkits*	509200
			Inventorycost	28283681	*Shelter*	318200
			Penaltycost	2643140	*Clothing*	254540

strong robustness compared to a deterministic solution. This robustness brings a certain price so that the robust solution can be conservative when it deals with the worst case scenario, which corresponds to maximum demand (nominal demand & perturbation demand). However, with a quite small risk, the box-polyhedral RC can effectively reduce the price of robustness.

5.2.3. Static vs. dynamic

In this section, we contrast the experimental results for the network in terms of the static and the dynamic commodity allocation models after a disaster. The static model (refer to Appendix B) simulates the standard approach, which assumes a single-period model. In other words, the static model does not take into account when and how the goods are transported to the affected area but only meets the balance between supply and demand. The dynamic model is the same as the nominal model with a one-fold penalty factor.

We will make a comparison between the static and dynamic models with the following assumptions. First, we assume that the two models are in a deterministic environment. Second, we assign the arc capacity of the static model to be infinite. Third, we set the penalty factor of the static model to be 5 times the purchase price of the commodities. Finally, we assume the same amount of demand at the candidate cities as in the dynamic model, i.e., the demand of each candidate city in the static model is equal to the accumulated demand for the period under the dynamic situation. The contrasting solutions given by the static and dynamic resource allocation formulations are summarized in Table 9.

We solve the total cost and penalty cost values of static and dynamic models using CPLEX commercial solver. The optimal objective values of the corresponding models are 1504.47 million CNY and 1533.15 million CNY, respectively. The static model and the dynamic model have similar optimal objective values. However, as shown in Table 9, the locations and sizes of facilities, as well as the various costs, are significantly different. The static model satisfies the balance of supply and demand, so the inventory and penalty costs of the static model are zero. Note that a static transportation network has been used to estimate the current and future allocation of emergency commodities. A static model can provide a better estimate

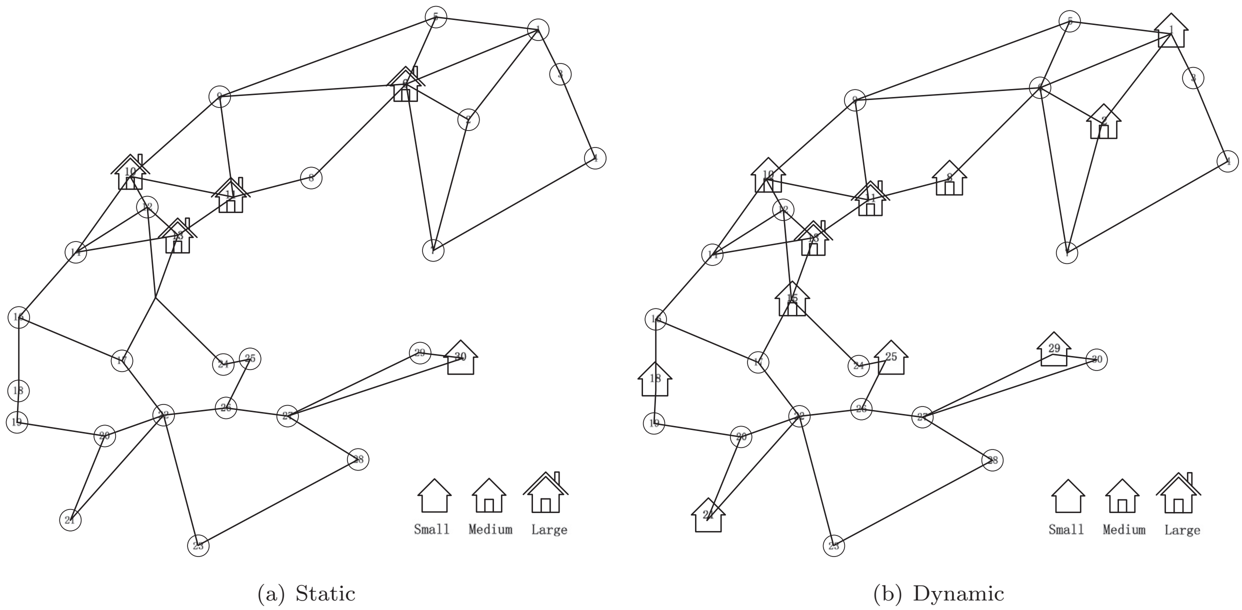


Fig. 7. Facility pre-positioning locations of static and dynamic models.

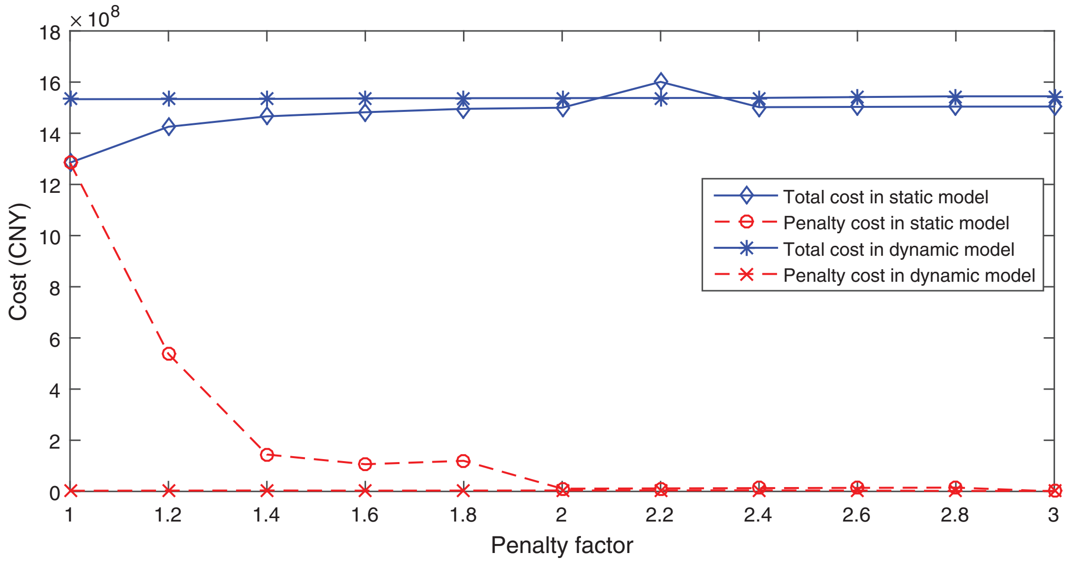


Fig. 8. Total cost and penalty cost of static and dynamic models with different penalty factors.

for the purpose of supply and demand balance. It represents the distribution of results without a time factor. However, the dynamic model requires that demand be met every day, or it will incur a penalty cost.

In reality, demand information can change over time based on changing conditions, which may have a significant impact on the response plan. The dynamic process is required about when and how many emergency commodities need to be transported. We apply the static model's solution to the dynamic model to calculate the results. Therefore, when the results of the static model suffer in a realistic environment, the response plans may become sub-optimal or even infeasible. With this policy, the violation value of the static model is 2361.64 million CNY, which increases losses by 57%. The pre-positioning locations of facilities in the static and dynamic models are shown in Fig. 7.

To test the sensitivity of the solution to the unmet demand penalty, we solve for the total cost and penalty cost with the static and dynamic models with different penalty factors (as shown in Fig. 8). The static model is very sensitive to changes in the penalty factor. When the penalty factor is small, there is a huge penalty. When the penalty factor is large, the penalty cost rapidly declines to zero. This is because the balance of supply and demand eliminates the penalty. However,

Table 10
Statistics of dynamic model with arc capacity constraint.

Facility size		Cost list(CNY)		Commodity(Unit)	
Large	11,13	*Fixedcost*	218359200	*Water*	76260
Medium	2,8,10,15	*Acquisitioncost*	1266985200	*Foodkits*	50800
Small	1,18,21,29	*Transportationcost*	5858821	*Medicalkits*	509200
		Inventorycost	27649791	*Shelter*	318200
		Penaltycost	3680977	*Clothing*	254540

this phenomenon is usually impractical in the real world. Although there are sufficient commodities, it is difficult to meet demand at every time point. For the dynamic model, the total cost and penalty cost are kept within a certain range. When the penalty factor is small, the cubic penalty function can also satisfy a certain amount of commodity demand.

It is also interesting to compare the models with and without arc capacity constraints. In this section, we obtain the solutions of a dynamic model with an arc capacity constraint (DWACCM). In addition, the solutions of a dynamic model without an arc capacity constraint (DOACCM) are summarized in Table 10. Compared with the entries of columns *Cost list* and *Commodity*, the two models have very similar costs and commodity quantities. However, there are some differences between the locations of the pre-positioning facilities in the two models. The solution of DWACCM uses the same large and medium locations as in the solution of DOACCM but adds a small facility at nodes 25 (DWACCM). Therefore, the solution constructed without the arc capacity constraint will not meet the situation give by the actual arc capacity constraint. In this case, a delay in the distribution of commodities will prevent the timely meeting of demand and increase the penalty cost from 3680977 CNY to 7016287 CNY. These changes are important to ensure that penalty costs are not caused by a lack of resources or by a delay in delivery.

6. Managerial implications

In this section, we will discuss and summarize some of the interesting managerial insights generated from our model formulation and case study analytical results.

(I) We formulate the pre-positioning of emergency supplies problem from the perspective of government organizations. Those investing in facilities and commodities wish to significantly reduce the loss of people's lives and property. The proposed model enables managers to make better decisions based on risk and cost factors. More specifically, this paper discusses the efficient planning of pre-positioning activities to provide the necessary disaster relief commodities to people in the affected areas immediately. The study of multi-period dynamic decisions can also provide a better overview of relief commodity needs and help managers to evaluate how these needs will be assigned over time.

(II) In our Circum-Bohai Sea Region case study, we first compare the distributionally robust model with the deterministic model. The experimental results show that the distributionally robust model obtains a relatively conservative solution. However, the solution is feasible for the randomly perturbed inequality with probability ϵ. In addition, under different penalty factors, the expected costs of the distributionally robust model have only minor changes, whereas the expected costs of the deterministic model show an upward trend. The distributionally robust model can adapt to a more urgent problem. Under the background of emergency supplies, managers are better off setting the penalty factor to a relatively high value.

(III) We also compare the distributionally robust model with the robust model. The solutions of the robust model (the box RC) usually make the constraint valid with probability 1, but this is too conservative. The expected cost of the box-polyhedral RC model is lower than that of the box RC model with risk degree $\epsilon = 0.05$. The expected cost of box-polyhedral RC reaches the minimum cost with the approximate risk $\epsilon = 0.009$. With a quite small risk, the box-polyhedral RC can effectively reduce the price of robustness. If the loss caused by the risk is serious, the managers should apply distributionally robust model to effectively reduce losses.

(IV) Finally, we contrast the experimental results for the network in terms of both the static and the dynamic commodity allocation model after a disaster, where the static commodity allocation model is a single-period model. A static model can provide a better estimate for the purpose of supply and demand balance. In practice, the implementation of the static model necessarily follows a dynamic process. By comparing the dynamic solution with the static solution substituted into the dynamic model, the violation value of the static model increases to 57% of losses. For a situation wherein demand changes over time, managers should pay more attention to multi-period dynamic decisions.

7. Conclusions

In this paper, a distributionally robust optimization model is proposed for the multi-period dynamic pre-positioning of emergency supplies problem with demand uncertainty. Our model is composed of two phases: the pre-disaster phase determines the inventory size and the location of facilities to open and the required inventory quantities of emergency commodities to purchase, and the dynamic post-disaster phase determines the routes and amount of transportation from the facilities to the affected areas. The aim of the proposed model is to minimize the total cost, including fixed, acquisition, transportation, holding and unmet demand penalties. In our model, time-variant demand is subject to uncertainty. A distributionally robust optimization approach is used to tackle the ambiguous chance constraints with box, box-ball and box-polyhedral perturbation sets. We obtain the computationally tractable robust counterparts (RCs) through the theorem of duality.

To illustrate the effectiveness of the distributionally robust optimization model, a realistic world application in the Circum-Bohai Sea Region of China is presented as an example. We first select 30 candidate cities according to the size of cities and the convenience of transportation in the Circum-Bohai Sea Region of China and connect them based on the actual railway distribution. Using the historical data from the USGS on the numbers of earthquakes greater than 4.0 during the last 50 years, we roughly estimate the probability that an earthquake will occur in each candidate city to obtain the nominal commodity demand. The resulting analysis demonstrates the robustness and conservatism of the proposed RC model. The distributionally robust optimization model outperforms the deterministic and robust optimization models for the same problem. In addition, the box RC is the most conservative model. The box-ball and box-polyhedral RC models can obtain a less conservative solution with a quite small risk. Analysis of this case study demonstrates the superiority of the multi-period dynamic pre-positioning of emergency supplies model, as the managers can dynamically assign the commodities based on the practical analysis of the post-disaster situation and changes in the confidence level.

Although there are an increasing number of studies in the field of disaster relief, there are still many limitations in this paper that need to be developed in future research. First, due to the large region, this study restricts transportation to railways only. Further research will be aimed at the case of the coexistence of railway and road transportation. Second, we assume that the rail network and depots would work after a major disaster. This assumption is reasonable because critical infrastructure should resist 7.0 Ms quakes. With an earthquake of magnitude 7 or more, railways and roads will be damaged, and helicopter rescue will be an important research topic. In this paper, we just focus on the transportation of railway without being damaged. Third, for simplicity, we restrict that the commodities require one day to be transported to a city. The relief time in distant cities usually tend to be slower than those in surrounding cities. In order to depict the time effective, the commodity transportation in non-adjacent cities will take more time. This assumption is also a limitation, and there may be other ways to depict this time effective, which will be an interesting research direction.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant No. 61773150 and 71801077), the Top-notch Talents of Hebei Province (Grant No. 702800118009), Social Science Foundation of Hebei Province (HB17GL012) and the High-Level Innovative Talent Foundation of Hebei University (Grant No. 521000981073).

Appendix A. Proofs

A.1. Proof of Proposition 1

Proof. For the ball perturbation set induced robust counterpart (17), the set can be denoted using conic representation

$$U_2 = \{\zeta \,|P_2\zeta + p_2 \in K_2\},$$

where

- $P_2\zeta = [\Sigma^{-1}\zeta; 0]$, $\Sigma = diag\{1, \ldots, 1\}$, $p_2 = [0_{L\times 1}; \Omega]$ and $K_2 = \{(z;t) \in R^L \times R : \|z\|_2 \le t\}$, whence $K_2^* = \{(z;t) \in R^L \times R : \|z\|_2 \le t\}$ □

Then the inner maximization problem in the constraint (14) can be denoted as

$$\max_\zeta \left\{ \sum_{t'=0}^{t-1} \zeta^t z^{t'} : P_2\zeta + p_2 \in K_2 \right\}. \tag{27}$$

Setting the dual variable $y = [\eta; \tau]$ with one-dimensional τ and L-dimensional η and using the dual cone K_2^*, the conic dual of (27) can be formulated as

$$\min_{\eta,\tau} \{\Omega\tau : \|\eta\|_2 \le \tau\}. \tag{28}$$

Then we can replace τ with $\|\eta\|_2 = \sqrt{\sum_{t'=0}^{t-1} (\eta^{t'})^2}$, and obtain $\Omega\sqrt{\sum_{t'=0}^{t-1} (\eta^{t'})^2}$.

Incorporating the above conic dual into the robust counterpart constraint, the following constraint is obtained

$$\varphi_{ik}^t + \Omega_{ik}^t \sqrt{\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2} \le 0, \quad \forall t \in T,\ i \in N,\ k \in K. \tag{29}$$

Based on the ball perturbation set, we induced the safe approximation of chance constraint (14)

$$\Pr\left\{ \varphi_{ik}^t + \sum_{t'=0}^{t-1} \zeta_{ik}^{t'} \psi_{ik}^{t'} > 0 \right\} \overset{(1)}{=} \Pr\left\{ \varphi_{ik}^t + \Omega_{ik}^t \sqrt{\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2} + \sum_{t'=0}^{t-1} \zeta_{ik}^{t'} \psi_{ik}^{t'} > \Omega_{ik}^t \sqrt{\sum_{t'=0}^{t-1} (\psi_{ik}^{t'})^2} \right\}$$

$$
\begin{aligned}
&\overset{(2)}{\le} \Pr\left\{\sum_{t'=0}^{t-1} \zeta_{ik}^{t'}\psi_{ik}^{t'} > \Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\psi_{ik}^{t'})^2}\right\} = \Pr\left\{\frac{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\psi_{ik}^{t'}}{\sqrt{\sum_{t'=0}^{t-1}(\psi_{ik}^{t'})^2}} > \Omega_{ik}^{t}\right\}\\
&\overset{(3)}{=} \Pr\left\{\sum_{t'=0}^{t-1} \zeta_{ik}^{t'}\alpha_{ik}^{t'} > \Omega_{ik}^{t}\right\} \overset{(4)}{\le} \Pr\left\{\sum_{t'=0}^{t-1} \zeta_{ik}^{t'}\alpha_{ik}^{t'} \ge \Omega_{ik}^{t}\right\} \overset{(5)}{=} \Pr\left\{\delta_{ik}^{t}\sum_{t'=0}^{t-1} \zeta_{ik}^{t'}\alpha_{ik}^{t'} \ge \delta_{ik}^{t}\Omega_{ik}^{t}\right\}\\
&\overset{(6)}{\le} E\left\{\exp\{\delta_{ik}^{t}\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\alpha_{ik}^{t'}\}\right\}\big/\exp\{\delta_{ik}^{t}\Omega_{ik}^{t}\} \overset{(7)}{=} \prod_{t'\in\{0,\dots t-1\}} E\left\{\exp\{\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'}\}\right\}\big/\exp\{\delta_{ik}^{t}\Omega_{ik}^{t}\}\\
&\overset{(8)}{=} \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}} E\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'})^n}{n!}\right\} = \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\int_{-1}^{1}\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'})^n}{n!}f(\zeta_{ik}^{t'})d\zeta_{ik}^{t'}\right\}\\
&\overset{(9)}{=} \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\int_{-1}^{1}\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'})^{2n}}{(2n)!}f(\zeta_{ik}^{t'})d\zeta_{ik}^{t'}\right\}\\
&\overset{(10)}{\le} \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\alpha_{ik}^{t'})^{2n}}{(2n)!}\right\} \overset{(11)}{=} \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\alpha_{ik}^{t'})^{2n}}{2^n n!}\right\}\\
&= \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\exp\{(\delta_{ik}^{t}\alpha_{ik}^{t'})^2/2\} = \exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\exp\{(\delta_{ik}^{t})^2\sum_{t'\in\{0,\dots t-1\}}(\alpha_{ik}^{t'})^2/2\}\\
&= \exp\{(\delta_{ik}^{t})^2/2 - \delta_{ik}^{t}\Omega_{ik}^{t}\}
\end{aligned}
$$

Note that in the above derivation, the equality (1) is obtained by adding $\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\psi_{ik}^{t'})^2}$ on both sides of the constraint; the inequality (2) is based on the inequality (29); in equality (3), $\alpha_{ik}^{t'} = \frac{\zeta_{ik}^{t'}\psi_{ik}^{t'}}{\sqrt{\sum_{t'=0}^{t-1}(\psi_{ik}^{t'})^2}}$; the equality (4) uses direct relaxation of the strict inequality; the equality (5) uses $\delta_{ik}^{t} > 0$; the equality (6) uses the Markov inequality; the equality (7) uses the independence condition; the equality (8) uses the Maclaurin series; the equality (9) uses the symmetric distribution condition; the equality (10) uses the $[-1, 1]$ bound condition; the equality (11) uses the fact $(2n)! > 2^n n!$. Finally, to obtain the best upper bound, let $\delta_{ik}^{t} = \Omega_{ik}^{t}$, the proof is complete. □

A.2. Proof of Theorem 1

Proof. Consider the box-ball perturbation set (19), then the set can be denoted using conic representation

$$U_{\infty\cap 2} = \{\zeta \,|\, P_\infty\zeta + p_\infty \in K_\infty, P_2\zeta + p_2 \in K_2\},$$

where

- $P_\infty\zeta = [\zeta; 0]$, $p_\infty = [0_{L\times 1}; \theta]$ and $K_\infty = \{(z;t) \in R^L \times R : \|z\|_\infty \le t\}$, whence $K_1^* = \{(z;t) \in R^L \times R : \|z\|_1 \le t\}$
- $P_2\zeta = [\Sigma^{-1}\zeta; 0]$, $\Sigma = diag\{1, \dots, 1\}$, $p_2 = [0_{L\times 1}; \Omega]$ and $K_2 = \{(z;t) \in R^L \times R : \|z\|_2 \le t\}$, whence $K_2^* = \{(z;t) \in R^L \times R : \|z\|_2 \le t\}$

Then the inner maximization problem in the constraint (14) can be denoted as

$$\max_\zeta \left\{\sum_{t'=0}^{t-1}\zeta^t z^{t'} : P_\infty\zeta + p_\infty \in K_\infty, P_2\zeta + p_2 \in K_2\right\}. \tag{30}$$

Setting the dual variable $y_1 = [\eta_1; \tau_1]$, $y_2 = [\eta_2; \tau_2]$ with one-dimensional τ_1, τ_2 and L-dimensional η_1, η_2 and using the dual cone K_1^*, K_2^*, the conic dual of (30) can be formulated as

$$\min_{\eta_1,\eta_2,\tau_1,\tau_2}\{\theta\tau_1 + \Omega\tau_2 : \eta_1 + \eta_2 = z, \|\eta_1\|_1 \le \tau_1, \|\eta_2\|_2 \le \tau_2\}. \tag{31}$$

Then we can replace τ_1 and τ_2 with $\|\eta_1\|_1 = \sum_{t'=0}^{t-1}|\eta_1^{t'}|$ and $\|\eta_2\|_2 = \sqrt{\sum_{t'=0}^{t-1}(\eta_2^{t'})^2}$, respectively, and obtain the variant of (31) as follows

$$
\begin{aligned}
&\min_{\eta_1,\eta_2}\left\{\theta\sum_{t'=0}^{t-1}|\eta_1^{t'}| + \Omega\sqrt{\sum_{t'=0}^{t-1}(\eta_2^{t'})^2} : \eta_1^{t'} + \eta_2^{t'} = z^{t'}\right\}\\
&= \theta\sum_{t'=0}^{t-1}|z^{t'} - \eta_2^{t'}| + \Omega\sqrt{\sum_{t'=0}^{t-1}(\eta_2^{t'})^2}.
\end{aligned} \tag{32}
$$

□

In addition, the above formulation can be transformed by introducing auxiliary variable $u^{t'}$ and constraint $-u^{t'} \le z^{t'} - \eta_2^{t'} \le u^{t'}$ as follows

$$\begin{cases} \theta \sum_{t'=0}^{t-1} u^{t'} + \Omega \sqrt{\sum_{t'=0}^{t-1} (\eta_2^{t'})^2} \\ -u^{t'} \le z^{t'} - \eta_2^{t'} \le u^{t'}. \end{cases} \tag{33}$$

Incorporating the conic dual (33) into the robust counterpart constraint, the following constraints are obtained

$$\begin{cases} \varphi_{ik}^t + \left[\sum_{t'=0}^{t-1} \theta_{ik}^{t'} u_{ik}^{t'} + \Omega_{ik}^t \sqrt{\sum_{t'=0}^{t-1} (\eta 1_{ik}^{t'})^2} \right] \le 0, & \forall t \in T,\ i \in N,\ k \in K, \\ -u_{ik}^{t'} \le \psi_{ik}^{t'} - \eta 1_{ik}^{t'} \le u_{ik}^{t'}, & \forall t \in T,\ t' \in \{0, \ldots, t-1\},\ i \in N,\ k \in K, \end{cases}$$

□

A.3. Proof of Theorem 2

Proof. Consider the box+polyhedral perturbation set (21), then the set can be denoted using conic representation

$$U_{\infty \cap 1} = \{\zeta | P_\infty \zeta + p_\infty \in K_\infty, P_1 \zeta + p_1 \in K_1\},$$

where

- $P_\infty \zeta = [\zeta; 0]$, $p_\infty = [0_{L \times 1}; \theta]$ and $K_\infty = \{(z;t) \in R^L \times R : \|z\|_\infty \le t\}$, whence $K_1^* = \{(z;t) \in R^L \times R : \|z\|_1 \le t\}$
- $P_1 \zeta = [\zeta; 0]$, $p_1 = [0_{L \times 1}; \Gamma]$ and $K_1 = \{(z;t) \in R^L \times R : \|z\|_2 \le t\}$, whence $K_\infty^* = \{(z;t) \in R^L \times R : \|z\|_\infty \le t\}$

Then the inner maximization problem in the constraint (16) can be denoted as

$$\max_\zeta \left\{ \sum_{t'=0}^{t-1} \zeta^t z^{t'} : P_\infty \zeta + p_\infty \in K_\infty, P_1 \zeta + p_1 \in K_1 \right\} \tag{34}$$

Setting the dual variable $y_1 = [\eta_1; \tau_1]$, $y_2 = [\eta_2; \tau_2]$ with one-dimensional τ_1, τ_2 and L-dimensional η_1, η_2 and using the dual cone K_1^*, K_∞^*, the conic dual of (34) can be formulated as

$$\min_{\eta_1, \eta_2, \tau_1, \tau_2} \{\theta \tau_1 + \Gamma \tau_2 : \eta_1 + \eta_2 = z, \|\eta_1\|_1 \le \tau_1, \|\eta_2\|_\infty \le \tau_2\} \tag{35}$$

Then we can replace τ_1 and τ_2 with $\|\eta_1\|_1 = \sum_{t'=0}^{t-1} |\eta_1^{t'}|$ and $\|\eta_2\|_\infty = max_{t' \in \{0,\ldots,t-1\}} |\eta_2^{t'}|$, respectively, and obtain the variant of (35) as follows

$$\min_{\eta_1, \eta_2} \left\{ \theta \sum_{t'=0}^{t-1} |\eta_1^{t'}| + \Gamma max_{t' \in \{0,\ldots,t-1\}} |\eta_2^{t'}| : \eta_1^{t'} + \eta_2^{t'} = z^{t'} \right\}$$

The above formulation is further equivalent to the following problem by introducing auxiliary variable u since it is a minimization problem

$$\min_{\eta_1, \eta_2} \left\{ \theta \sum_{t'=0}^{t-1} |\eta_1^{t'}| + \Gamma u : u \ge |z^{t'} - \eta_1^{t'}|,\ \forall t' \in \{0, \ldots, t-1\} \right\}$$

Realizing that $|z^{t'} - \eta_1^{t'}| \ge |z^{t'}| - |\eta_1^{t'}|$ and $z^{t'}, \eta_1^{t'} \ge 0$, we can get an equivalent problem as follows

$$\min_{\eta_1, \eta_2} \left\{ \theta \sum_{t'=0}^{t-1} \eta_1^{t'} + \Gamma u : u \ge z^{t'} - \eta_1^{t'},\ \forall t' \in \{0, \ldots, t-1\}, \eta_1^{t'} \ge 0, u \ge 0 \right\} \tag{36}$$

Incorporating the conic dual (36) into the robust counterpart constraint, the following constraints are obtained

$$\begin{cases} \varphi_{ik}^t + \left[\sum_{t'=0}^{t-1} \theta_{ik}^{t'} \eta 1_{ik}^{t'} + \Gamma_{ik}^t u_{ik}^t \right] \le 0, & \forall t \in T,\ i \in N,\ k \in K, \\ \eta 1_{ik}^{t'} + u_{ik}^t \ge \psi_{ik}^{t'}, & \forall t \in T,\ t' \in \{0, \ldots, t-1\},\ i \in N,\ k \in K \\ \eta 1_{ik}^{t'}, u_{ik}^t \ge 0, & \forall t \in T,\ t' \in \{0, \ldots, t-1\},\ i \in N,\ k \in K, \end{cases}$$

□

A.4. Proof of Theorem 3

Proof. Based on the box-ball perturbation set, we induced the safe approximation of chance constraint (14)

$$\begin{aligned}
&\Pr\left\{\varphi_{ik}^{t}+\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\psi_{ik}^{t'}>0\right\}\\
&\overset{(1)}{=}\Pr\left\{\varphi_{ik}^{t}+\left[\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}(\psi_{ik}^{t'}-\eta 1_{ik}^{t'})+\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}\right]+\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\eta 1_{ik}^{t'}>\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}\right\}\\
&\overset{(2)}{\le}\Pr\left\{\varphi_{ik}^{t}+\left[\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}|\psi_{ik}^{t'}-\eta 1_{ik}^{t'}|+\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}\right]+\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\eta 1_{ik}^{t'}>\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}\right\}\\
&\overset{(3)}{\le}\Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\eta 1_{ik}^{t'}>\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}\right\}=\Pr\left\{\frac{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\eta 1_{ik}^{t'}}{\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}}>\Omega_{ik}^{t}\right\}\\
&\overset{(4)}{=}\Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\alpha_{ik}^{t'}>\Omega_{ik}^{t}\right\}\overset{(5)}{\le}\Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\alpha_{ik}^{t'}\ge\Omega_{ik}^{t}\right\}\overset{(6)}{=}\Pr\left\{\delta_{ik}^{t}\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\alpha_{ik}^{t'}\ge\delta_{ik}^{t}\Omega_{ik}^{t}\right\}\\
&\overset{(7)}{\le}E\left\{\exp\{\delta_{ik}^{t}\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\alpha_{ik}^{t'}\}\right\}\Big/\exp\{\delta_{ik}^{t}\Omega_{ik}^{t}\}\overset{(8)}{=}\prod_{t'\in\{0,\dots t-1\}}E\left\{\exp\{\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'}\}\right\}\Big/\exp\{\delta_{ik}^{t}\Omega_{ik}^{t}\}\\
&\overset{(9)}{=}\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}E\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'})^n}{n!}\right\}=\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\int_{-\theta_{ik}^{t'}}^{\theta_{ik}^{t'}}\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'})^n}{n!}f(\zeta_{ik}^{t'})d\zeta_{ik}^{t'}\right\}\\
&\overset{(10)}{=}\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\int_{-\theta_{ik}^{t'}}^{\theta_{ik}^{t'}}\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\alpha_{ik}^{t'})^{2n}}{(2n)!}f(\zeta_{ik}^{t'})d\zeta_{ik}^{t'}\right\}\\
&\overset{(11)}{\le}\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\theta_{ik}^{t'}\alpha_{ik}^{t'})^{2n}}{(2n)!}\right\}\overset{(12)}{=}\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\theta_{ik}^{t'}\alpha_{ik}^{t'})^{2n}}{2^n n!}\right\}\\
&=\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\prod_{t'\in\{0,\dots t-1\}}\exp\{(\delta_{ik}^{t}\theta_{ik}^{t'}\alpha_{ik}^{t'})^2/2\}=\exp\{-\delta_{ik}^{t}\Omega_{ik}^{t}\}\exp\{(\delta_{ik}^{t})^2\sum_{t'\in\{0,\dots t-1\}}(\theta_{ik}^{t'}\alpha_{ik}^{t'})^2/2\}\\
&\overset{(13)}{=}\exp\{(\theta_{ik}^{t}\delta_{ik}^{t})^2/2-\delta_{ik}^{t}\Omega_{ik}^{t}\}
\end{aligned}$$

□

Note that in the above derivation, the equality (1) is obtained by adding $\Omega_{ik}^{t}\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}$ on both sides of the constraint; the inequality (2) uses absolute value relaxation; the inequality (3) is based on the box-ball perturbation set (19) and formula (20); in equality (4), $\alpha_{ik}^{t'}=\frac{\eta 1_{ik}^{t'}}{\sqrt{\sum_{t'=0}^{t-1}(\eta 1_{ik}^{t'})^2}}$. Then, we have $\sum_{t'=0}^{t-1}\alpha_{ik}^{t'2}=1$; the equality (5) uses direct relaxation of the strict inequality; the equality (6) uses $\delta_{ik}^{t}>0$; the equality (7) uses the Markov inequality; the equality (8) uses the independence condition; the equality (9) uses the Maclaurin series; the equality (10) uses the symmetric distribution condition; the equality (11) uses the $[-\theta_{ik}^{t'},\theta_{ik}^{t'}]$ bound condition; the equality (12) uses the fact $(2n)!>2^n n!$; in equality (13), $\theta_{ik}^{t2}=\sum_{t'=0}^{t-1}(\theta_{ik}^{t'}\alpha_{ik}^{t'})^2$. Finally, to obtain the best upper bound, let $\delta_{ik}^{t}=\frac{\Omega_{ik}^{t}}{(\theta_{ik}^{t})^2}$, the proof is complete. □

A.5. Proof of Theorem 4

Proof. Based on the box-polyhedral perturbation set, we induced the safe approximation of chance constraint (14)

$$\begin{aligned}
&\Pr\left\{\varphi_{ik}^{t}+\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\psi_{ik}^{t'}>0\right\}=\Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\psi_{ik}^{t'}>-\varphi_{ik}^{t}\right\}\\
&\overset{(1)}{\le}\Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\psi_{ik}^{t'}>\left[\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\eta 1_{ik}^{t'}+\Gamma_{ik}^{t}u_{ik}^{t}\right]\right\}
\end{aligned}$$

$$\overset{(2)}{\leq} \Pr\left\{\sum_{t'=0}^{t-1} \zeta_{ik}^{t'}(\psi_{ik}^{t'} - \eta 1_{ik}^{t'}) > \left[\Gamma_{ik}^{t} \max_{t'\in\{0,\dots t-1\}}(\psi_{ik}^{t'} - \eta 1_{ik}^{t'})\right]\right\} = \Pr\left\{\frac{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}(\psi_{ik}^{t'} - \eta 1_{ik}^{t'})}{\max_{t'\in\{0,\dots t-1\}}(\psi_{ik}^{t'} - \eta 1_{ik}^{t'})} > \Gamma_{ik}^{t}\right\}$$
$$\overset{(3)}{=} \Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\beta_{ik}^{t'} > \Gamma_{ik}^{t}\right\} \overset{(4)}{\leq} \Pr\left\{\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\beta_{ik}^{t'} \geq \Gamma_{ik}^{t}\right\} \overset{(5)}{=} \Pr\left\{\delta_{ik}^{t}\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\beta_{ik}^{t'} \geq \delta_{ik}^{t}\Gamma_{ik}^{t}\right\}$$
$$\overset{(6)}{\leq} E\left\{\exp\{\delta_{ik}^{t}\sum_{t'=0}^{t-1}\zeta_{ik}^{t'}\beta_{ik}^{t'}\}\right\} \Big/ \exp\{\delta_{ik}^{t}\Gamma_{ik}^{t}\} \overset{(7)}{=} \prod_{t'\in\{0,\dots t-1\}} E\left\{\exp\{\delta_{ik}^{t}\zeta_{ik}^{t'}\beta_{ik}^{t'}\}\right\} \Big/ \exp\{\delta_{ik}^{t}\Gamma_{ik}^{t}\}$$
$$\overset{(8)}{=} \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\} \prod_{t'\in\{0,\dots t-1\}} E\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\beta_{ik}^{t'})^{n}}{n!}\right\} = \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\} \prod_{t'\in\{0,\dots t-1\}}\left\{\int_{-1}^{1}\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\beta_{ik}^{t'})^{n}}{n!} f(\zeta_{ik}^{t'})d\zeta_{ik}^{t'}\right\}$$
$$\overset{(9)}{=} \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\} \prod_{t'\in\{0,\dots t-1\}}\left\{\int_{-1}^{1}\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\zeta_{ik}^{t'}\beta_{ik}^{t'})^{2n}}{(2n)!} f(\zeta_{ik}^{t'})d\zeta_{ik}^{t'}\right\}$$
$$\overset{(10)}{\leq} \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\} \prod_{t'\in\{0,\dots t-1\}}\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\theta_{ik}^{t'}\beta_{ik}^{t'})^{2n}}{(2n)!}\right\} \overset{(11)}{=} \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\} \prod_{t'\in\{0,\dots t-1\}}\left\{\sum_{n=0}^{\infty}\frac{(\delta_{ik}^{t}\theta_{ik}^{t'}\beta_{ik}^{t'})^{2n}}{2^{n}n!}\right\}$$
$$= \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\} \prod_{t'\in\{0,\dots t-1\}} \exp\{(\delta_{ik}^{t}\theta_{ik}^{t'}\beta_{ik}^{t'})^{2}/2\} = \exp\{-\delta_{ik}^{t}\Gamma_{ik}^{t}\}\exp\{(\delta_{ik}^{t})^{2}\sum_{t'\in\{0,\dots t-1\}}(\theta_{ik}^{t'}\beta_{ik}^{t'})^{2}/2\}$$
$$\overset{(12)}{\leq} \exp\{t(\theta_{ik}^{t}\delta_{ik}^{t})^{2}/2 - \delta_{ik}^{t}\Gamma_{ik}^{t}\} \qquad \square$$

Note that in the above derivation, the equality (1) is obtained based on the box-polyhedral perturbation set (21) and the first constraint of the formulas (22); the inequality (2) is obtained based on the second constraint of the formulas (22); in equality (3), $\beta_{ik}^{t'} = \frac{\psi_{ik}^{t'} - \eta 1_{ik}^{t'}}{\max_{t'\in\{0,\dots t-1\}}(\psi_{ik}^{t'} - \eta 1_{ik}^{t'})}$. Then, we have $\beta_{ik}^{t'} \leq 1, \forall t' \in \{0,\dots t-1\}$; the equality (4) uses direct relaxation of the strict inequality; the equality (5) uses $\delta_{ik}^{t} > 0$; the equality (6) uses the Markov inequality; the equality (7) uses the independence condition; the equality (8) uses the Maclaurin series; the equality (9) uses the symmetric distribution condition; the equality (10) uses the $[-\theta_{ik}^{t'}, \theta_{ik}^{t'}]$ bound condition; the equality (11) uses the fact $(2n)! > 2^{n}n!$; in equality (12), $t\theta_{ik}^{t} = \sum_{t'=0}^{t-1}\theta_{ik}^{t'}\beta_{ik}^{t'}$, where t is the dimension of $\beta_{ik}^{t'}$. Finally, to obtain the best upper bound, let $\delta_{ik}^{t} = \frac{\Gamma_{ik}^{t}}{t(\theta_{ik}^{t})^{2}}$, the proof is complete.

Appendix B. Deterministic model, box-polyhedral RC model and static model

B.1 Deterministic model

$$\begin{aligned}
\min_{y,q,x,z,w} \quad & \sum_{i\in N}\sum_{l\in L}F_{il}y_{il} + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K}a_k q_{ik}^{t} + \sum_{t\in T}\sum_{(i,j)\in A}\sum_{k\in K}c_k r_{ij}x_{ijk}^{t} + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K}(h_k z_{ik}^{t} + M_k^{t}w_{ik}^{t}) \\
\text{s.t.} \quad & w_{ik}^{t} - z_{ik}^{t} = \sum_{t'=0}^{t-1}\left(\sum_{(i,j)\in A}x_{ijk}^{t'} - \sum_{(j,i)\in A}x_{jik}^{t'} + \bar{d}_{ik}^{t'} - q_{ik}^{t'}\right), \quad \forall t\in T,\ i\in N,\ k\in K, \\
& \sum_{t\in T}\sum_{k\in K}v_k q_{ik}^{t} \leq \sum_{l\in L}\gamma_l y_{il}, \quad \forall i\in N, \\
& \sum_{k\in K}b^{k}x_{ijk}^{t} \leq B_{ij}, \quad \forall t\in T,\ (i,j)\in A, \\
& \sum_{l\in L}y_{il} \leq 1, \quad \forall i\in N, \\
& \sum_{(i,j)\in A}x_{ijk}^{t} \leq z_{ik}^{t} - w_{ik}^{t}, \quad \forall t\in T,\ i\in N,\ k\in K, \\
& y_{il} \in (0,1), \quad \forall i\in N,\ l\in L, \\
& q_{ik}^{t}, w_{ik}^{t} \geq 0, \quad \forall t\in T,\ i\in N,\ k\in K, \\
& x_{ijk}^{t} \geq 0, \quad \forall t\in T,\ (i,j)\in A,\ k\in K.
\end{aligned}$$

B.2 Box-polyhedral RC model

$$
\begin{aligned}
\min_{y,q,x,z,w} \quad & \sum_{i\in N}\sum_{l\in L} F_{il}y_{il} + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K} a_k q_{ik}^t + \sum_{t\in T}\sum_{(i,j)\in A}\sum_{k\in K} c_k r_{ij} x_{ijk}^t + \sum_{t\in T}\sum_{i\in N}\sum_{k\in K}(h_k z_{ik}^t + M_k^t w_{ik}^t) \\
\text{s. t.} \quad & \sum_{t'=0}^{t-1}\left(\sum_{(i,j)\in A} x_{ijk}^{t'} - \sum_{(j,i)\in A} x_{jik}^{t'} + \bar{d}_{ik}^{t'} - q_{ik}^{t'}\right) + \sum_{t'=0}^{t-1}\theta_{ik}^{t'}\eta 1_{ik}^{t'} + \Gamma_{ik}^t u_{ik}^t \le w_{ik}^t - z_{ik}^t, \quad \forall t\in T,\ i\in N,\ k\in K, \\
& \eta 1_{ik}^{t'} + u_{ik}^t \ge \hat{d}_{ik}^{t'}, \quad \forall t\in T,\ t'\in\{0,\dots,t-1\},\ i\in N,\ k\in K, \\
& \sum_{t\in T}\sum_{k\in K} v_k q_{ik}^t \le \sum_{l\in L}\gamma_l y_{il}, \quad \forall i\in N, \\
& \sum_{k\in K} b^k x_{ijk}^t \le B_{ij}, \quad \forall t\in T,\ (i,j)\in A, \\
& \sum_{l\in L} y_{il} \le 1, \quad \forall i\in N, \\
& \sum_{(i,j)\in A} x_{ijk}^t \le z_{ik}^t - w_{ik}^t, \quad \forall t\in T,\ i\in N,\ k\in K, \\
& y_{il}\in(0,1), \quad \forall i\in N,\ l\in L, \\
& q_{ik}^t, w_{ik}^t \ge 0, \quad \forall t\in T,\ i\in N,\ k\in K, \\
& x_{ijk}^t \ge 0, \quad \forall t\in T,\ (i,j)\in A,\ k\in K, \\
& \eta 1_{ik}^{t'}, u_{ik}^t \ge 0, \quad \forall t\in T,\ t'\in\{0,\dots,t-1\},\ i\in N,\ k\in K.
\end{aligned}
$$

B.3 Static model

When the period T is not considered, the static model can be formulated as

$$
\begin{aligned}
\min_{y,q,x,z,w} \quad & \sum_{i\in N}\sum_{l\in L} F_{il}y_{il} + \sum_{i\in N}\sum_{k\in K} a_k q_{ik} + \sum_{(i,j)\in A}\sum_{k\in K} c_k r_{ij} x_{ijk} + \sum_{i\in N}\sum_{k\in K}(h_k z_{ik} + M_k w_{ik}) \\
\text{s. t.} \quad & \sum_{(j,i)\in A} x_{jik} + q_{ik} - z_{ik} = \sum_{(i,j)\in A} x_{ijk} + d_{ik} - w_{ik}, \quad \forall i\in N,\ k\in K, \\
& \sum_{k\in K} v_k q_{ik} \le \sum_{l\in L}\gamma_l y_{il}, \quad \forall i\in N, \\
& \sum_{k\in K} b^k x_{ijk} \le B_{ij}, \quad \forall (i,j)\in A, \\
& \sum_{l\in L} y_{il} \le 1, \quad \forall i\in N, \\
& y_{il}\in(0,1), \quad \forall i\in N,\ l\in L, \\
& q_{ik}, w_{ik} \ge 0, \quad \forall i\in N,\ k\in K, \\
& x_{ijk} \ge 0, \quad \forall (i,j)\in A,\ k\in K.
\end{aligned}
$$

References

[1] N. Altay, W.G. Green, OR/MS research in disaster operations management, Eur. J. Oper. Res. 175 (1) (2006) 475–493.
[2] G. Galindo, R. Batta, Review of recent developments in OR/MS research in disaster operations management, Eur. J. Oper. Res. 230 (2) (2013) 201–211.
[3] C.G. Rawls, M.A. Turnquist, Pre-positioning and dynamic delivery planning for short-term response following a natural disaster, Socio-Econ. Plan. Sci. 46 (1) (2012) 46–54.
[4] G. Kovács, K.M. Spens, Humanitarian logistics in disaster relief operations, Int. J. Phys. Distrib. Logist. Manag. 37 (2) (2007) 99–114.
[5] J.V. Florez, M. Lauras, U. Okongwu, L. Dupont, A decision support system for robust humanitarian facility location, Eng. Appl. Artif. Intell. 46 (2015) 326–335.
[6] S.A. Torabi, I. Shokr, S. Tofighi, J. Heydari, Integrated relief pre-positioning and procurement planning in humanitarian supply chains, Transp. Res. Part E: Logist. Transp. Rev. 113 (2018) 123–146.
[7] P. Hasanov, M.Y. Jaber, N. Tahirov, Four-level closed loop supply chain with remanufacturing, Appl. Math. Model. 66 (2019) 141–155.
[8] C.G. Rawls, M.A. Turnquist, Pre-positioning of emergency supplies for disaster response, Transport. Res. Part B: Methodol. 44 (4) (2010) 521–534.
[9] H. Hasanzadeh, M. Bashiri, An efficient network for disaster management: Model and solution, Appl. Math. Model. 40 (2016) 3688–3702.
[10] R. Mohammadi, S.M.T.F. Ghomi, F. Jolai, Prepositioning emergency earthquake response supplies: A new multi-objective particle swarm optimization algorithm, Appl. Math. Model. 40 (9–10) (2016) 5183–5199.
[11] S. Tofighi, S.A. Torabi, S.A. Mansouri, Humanitarian logistics network design under mixed uncertainty, Eur. J. Oper. Res. 250 (1) (2016) 239–250.
[12] M. Ahmadi, A. Seifi, B. Tootooni, A humanitarian logistics model for disaster relief operation considering network failure and standard relief time: A case study on san francisco district, Transp. Res. Part E: Logist. Transp. Rev. 75 (2015) 145–163.
[13] G. Barbarosoglu, L. Ozdamar, A. Cevik, An interactive approach for hierarchical analysis of helicopter logistics in disaster relief operations, Eur. J. Oper. Res. 140 (1) (2002) 118–133.
[14] Y. Liu, H. Lei, D. Zhang, Z. Wu, Robust optimization for relief logistics planning under uncertainties in demand and transportation time, Appl. Math. Model. 55 (2018) 262–280.
[15] A.M. Caunhye, X. Nie, S. Pokharel, Optimization models in emergency logistics: A literature review, Socio-Econ. Plan. Sci. 46 (1) (2012) 4–13.
[16] J. Acimovic, J. Goentzel, Models and metrics to assess humanitarian response capacity, J. Oper. Manag. 45 (2016) 11–29.
[17] Y. Zhou, J.D. Liu, Y. Zhang, X. Gan, A multi-objective evolutionary algorithm for multi-period dynamic emergency resource scheduling problems, Transp. Res. Part E: Logist. Transp. Rev. 99 (2017) 77–95.

[18] R.A. Garrett, T.C. Sharkey, M. Grabowski, W.A. Wallace, Dynamic resource allocation to support oil spill response planning for energy exploration in the arctic, Eur. J. Oper. Res. 257 (1) (2017) 272–286.
[19] N. Kunz, G. Reiner, S. Gold, Investing in disaster management capabilities versus pre-positioning inventory: A new approach to disaster preparedness, Int. J. Prod. Econ. 157 (2014) 261–272.
[20] W. Yi, L. Özdamar, A dynamic logistics coordination model for evacuation and support in disaster response activities, Eur. J. Oper. Res. 179 (3) (2007) 1177–1193.
[21] A. Bozorgi-Amiri, M. Khorsi, A dynamic multi-objective location-routing model for relief logistic planning under uncertainty on demand, travel time, and cost parameters, Int. J. Adv Manuf. Technol. 70 (5-8) (2016) 1633–1648. 85(5–8)
[22] D.A. Serel, A single-period stocking and pricing problem involving stochastic emergency supply, Int. J. Prod. Econ. 185 (2017) 180–195.
[23] A. Charles, M. Lauras, L.N.V. Wassenhove, L. Dupont, Designing an efficient humanitarian supply network, J. Oper. Manag. 47 (2016) 58–70.
[24] R. Wang, K. Yang, L. Yang, Designing hub-and-spoke network with uncertain travel times: A new hybrid methodology, J. Uncertain Syst. (2017) 243–256.
[25] M. Yang, G. Yang, Robust optimization for the single allocation p-hub median problem under discount factor uncertainty, J. Uncertain Syst. 11 (3) (2017) 230–240.
[26] A. Bozorgi-Amiri, M.S. Jabalameli, S.M.J.M. Al-e Hashem, A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty, OR Spectr. 35 (4) (2013) 905–933.
[27] J. Salmerón, A. Apte, Stochastic optimization for natural disaster asset prepositioning, Prod. Oper. Manag. 19 (5) (2010) 561–574.
[28] A. Mohamadi, S. Yaghoubi, A bi-objective stochastic model for emergency medical services network design with backup services for disasters under disruptions: An earthquake case study, Int.J. Disaster Risk Reduct. 23 (2017) 204–217.
[29] C.H. Glock, K. Schwindl, M.Y. Jaber, An EOQ model with fuzzy demand and learning in fuzziness, Int. J. Serv. Oper. Manag. 12 (1) (2012) 90–100.
[30] X. Bai, J. Gao, Y. Liu, Prepositioning emergency supplies under uncertainty: A parametric optimization method, Eng. Optim. 50 (7) (2018) 1114–1133.
[31] D. Bertsimas, M. Sim, The price of robustness, Oper. Res. 52 (1) (2004) 35–53.
[32] A. Ben-Tal, L.E. Ghaoui, A. Nemirovski, Robust Optimization, Princeton University Press, New Jersey, 2009.
[33] A.S. Safaei, S. Farsad, M.M. Paydar, Robust bi-level optimization of relief logistics operations, Appl. Math. Model. 56 (2018) 359–380.
[34] S. Zokaee, A. Bozorgi-Amiri, S.J. Sadjadi, A robust optimization model for humanitarian relief chain design under uncertainty, Appl. Math. Model. 40 (17–18) (2016) 7996–8016.
[35] C.L. Hu, X. Liu, Y.K. Hua, A bi-objective robust model for emergency resource allocation under uncertainty, Int. J. Produc. Res. 54 (24) (2016) 7421–7438.
[36] M. Haghi, S.M.T.F. Ghomi, F. Jolai, Developing a robust multi-objective model for pre/post disaster times under uncertainty in demand and resource, J. Clean. Prod. 154 (2017) 188–202.
[37] M. Najafi, K. Eshghi, W. Dullaert, A multi-objective robust optimization model for logistics planning in the earthquake response phase, Transp. Res. Part E: Logist. Transp. Rev 49 (1) (2013) 217–249.
[38] A. Ben-Tal, B.D. Chung, S.R. Mandala, T. Yao, Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains, transport, Res. Part B: Methodol. 45 (8) (2011) 1177–1189.
[39] W. Ni, J. Shu, M. Song, Location and emergency inventory pre-positioning for disaster response operations: Min-max robust model and a case study of yushu earthquake, Prod. Oper. Manag. 27 (1) (2018) 160–183.
[40] J. Goh, M. Sim, Distributionally robust optimization and its tractable approximations, Oper. Res. 58 (2010) 902–917. 4-part-1
[41] E. Delage, Y. Ye, Distributionally robust optimization under moment uncertainty with application to data-driven problems, Oper. Res. 58 (3) (2010) 595–612.
[42] W. Wiesemann, D. Kuhn, M. Sim, Distributionally robust convex optimization, Oper. Res. 62 (6) (2014) 1358–1376.
[43] G.A. Hanasusanto, D. Kuhn, S.W. Wallace, S. Zymler, Distributionally robust multi-item newsvendor problems with multimodal demand distributions, Math. Program. 152 (1–2) (2015) 1–32.
[44] Y. Zhang, S. Shen, J.L. Mathieu, Distributionally robust chance-constrained optimal power flow with uncertain renewables and uncertain reserves provided by loads, IEEE Trans. Power Syst. 32 (2) (2017) 1378–1388.
[45] A. Nemirovski, A. Shapiro, Convex approximations of chance constrained programs, SIAM J. Optim. 17 (4) (2006) 969–996.
[46] R. Jiang, Y. Guan, Risk-averse two-stage stochastic program with distributional ambiguity, Oper. Res. 66 (5) (2018) 1390–1405.
[47] D. Bertsimas, M. Sim, M. Zhang, Adaptive distributionally robust optimization, Manag. Sci. (2018), doi:10.1287/mnsc.2017.2952.
[48] M. Wen, J.F. Cordeau, G. Laporte, J. Larsen, The dynamic multi-period vehicle routing problem, Comput. Oper. Res. 37 (9) (2010) 1615–1623.
[49] K.F. Doerner, W.J. Gutjahr, P.C. Nolz, Multi-criteria location planning for public facilities in tsunami-prone coastal areas, OR Spectr. 31 (3) (2009) 651–678.
[50] M. Wyss, S. Wiemer, Change in the probability for earthquakes in southern california due to the landers magnitude 7.3 earthquake, Sci. 290 (5495) (2000) 1334–1338.
[51] Sphere Project, Humanitarian charter and minimum standards in humanitarian response, Practical Action Publishing, 2011.
[52] M. Gama, B.F. Santos, M.P. Scaparra, A multi-period shelter location-allocation model with evacuation orders for flood disasters, EURO J. Comput. Optimi. 4 (3-4) (2016) 299–323.
[53] A. Coburn, R. Spence, A. Pomonis, Factors determining human casualty levels in earthquakes: Mortality prediction in building collapse, The tenth world conference on earthquake engineering (1992). Rotterdam, Netherlands